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Structural Steel Design
Compression Members

Faculty of Engineering & Applied Science

Columns in Buildings

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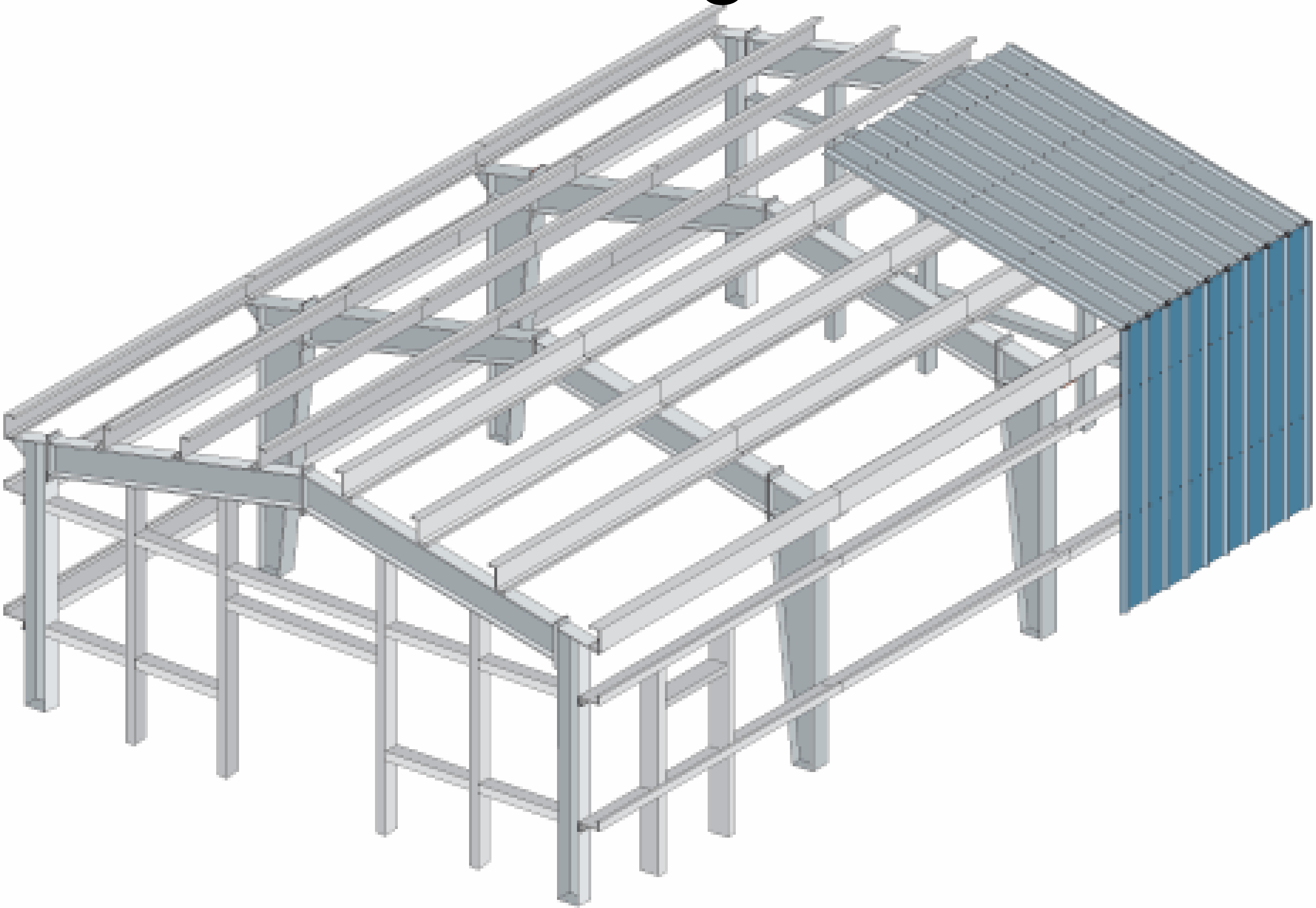
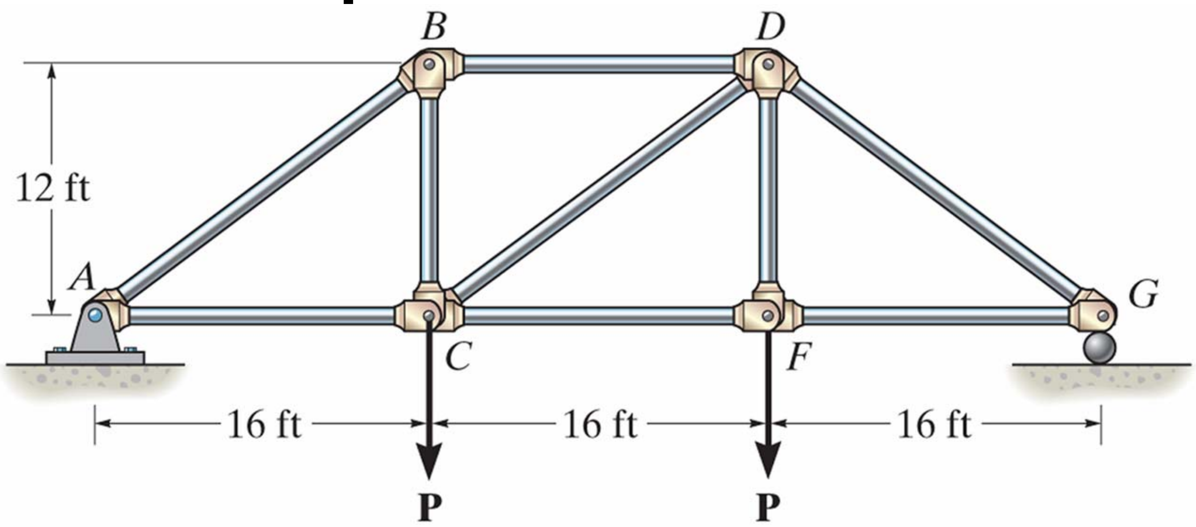
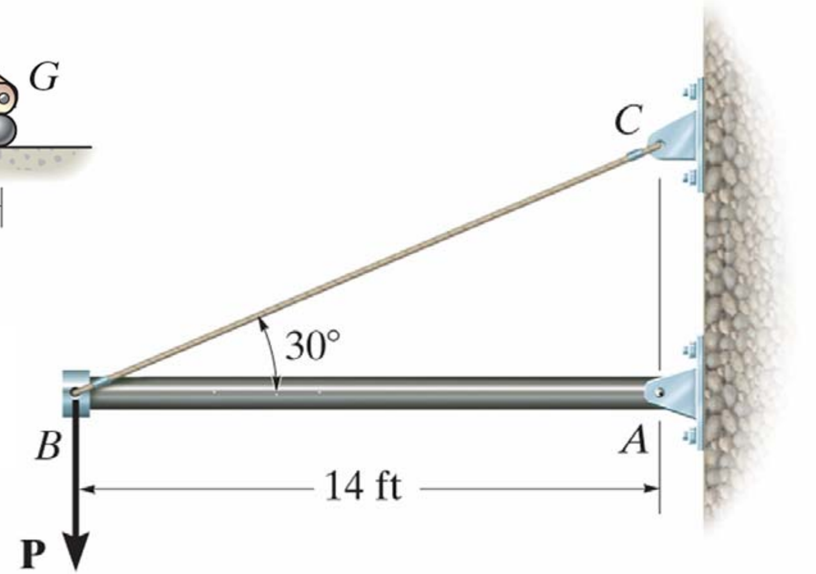
Columns in Buildings

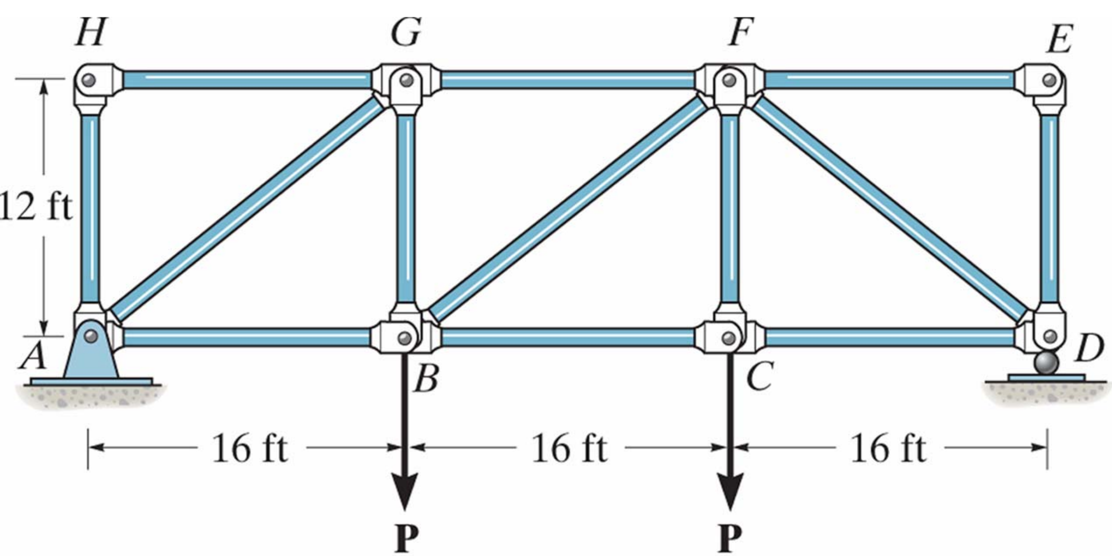


Column supports

Compression members in trusses







Compression members in trusses

Compression members in OWSJ

Compression members in bridges

Howrah bridge, Kolkata, India

Compression members in towers

Eiffel Tower (1887 - 89)

The new Tokyo Tower is set to be completed in 2011. It will stand 610m high.

Compression in equipment

Introduction

- **Steel Compression members**
  - Building columns
  - Frame Bracing
  - Truss members (chords and bracing)
- Useful in pure compression as well as in beam-columns
- **Design Clauses: CAN/CSA-S16**
  - Over-all strength as per Clause 13.3
  - Local buckling check: Clause 11 (Table 1)
  - Built-up members: Clause 19

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Column erection

Different column c/s shapes

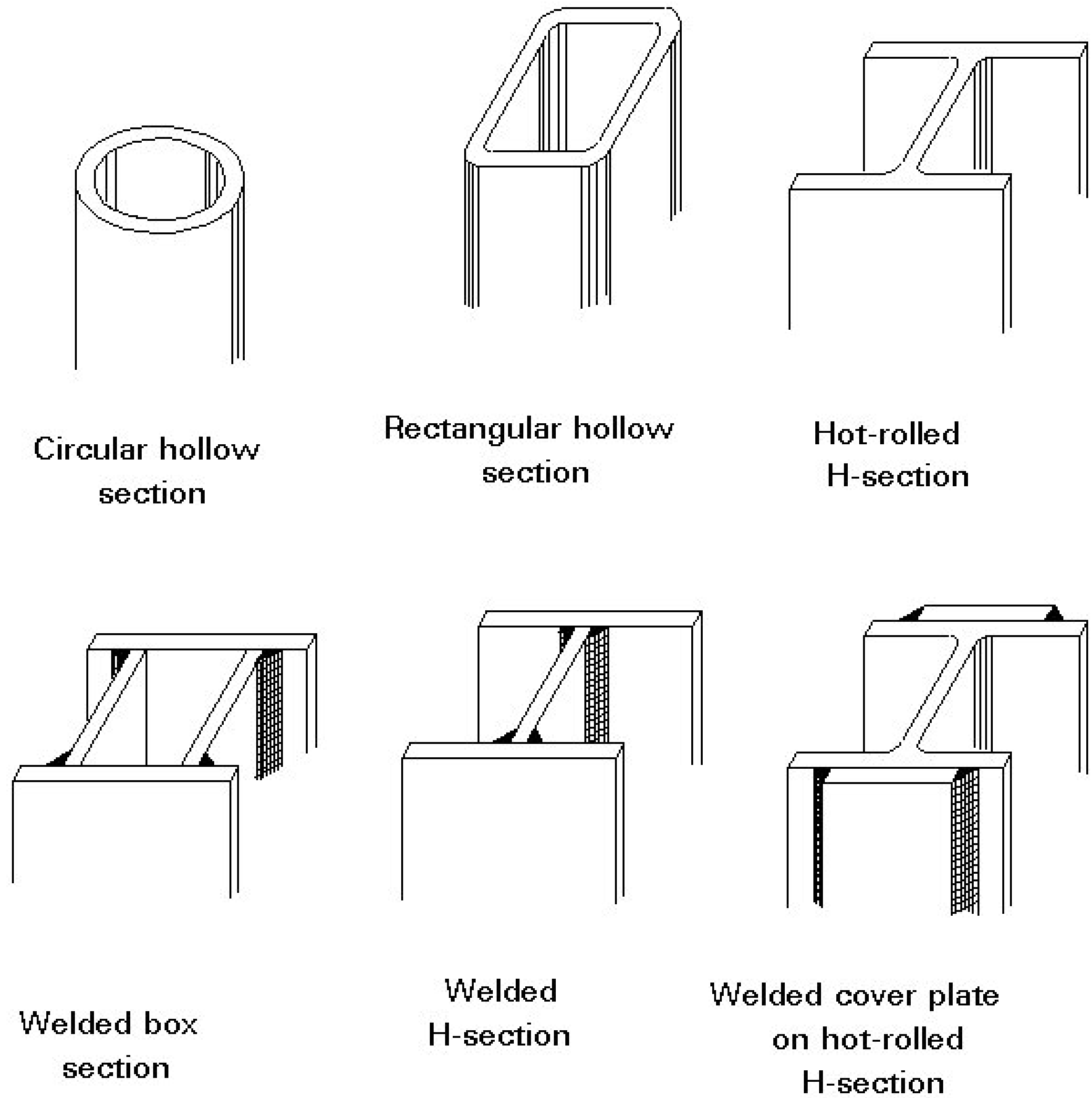


Figure 1 Simple compression members

Different column c/s shapes

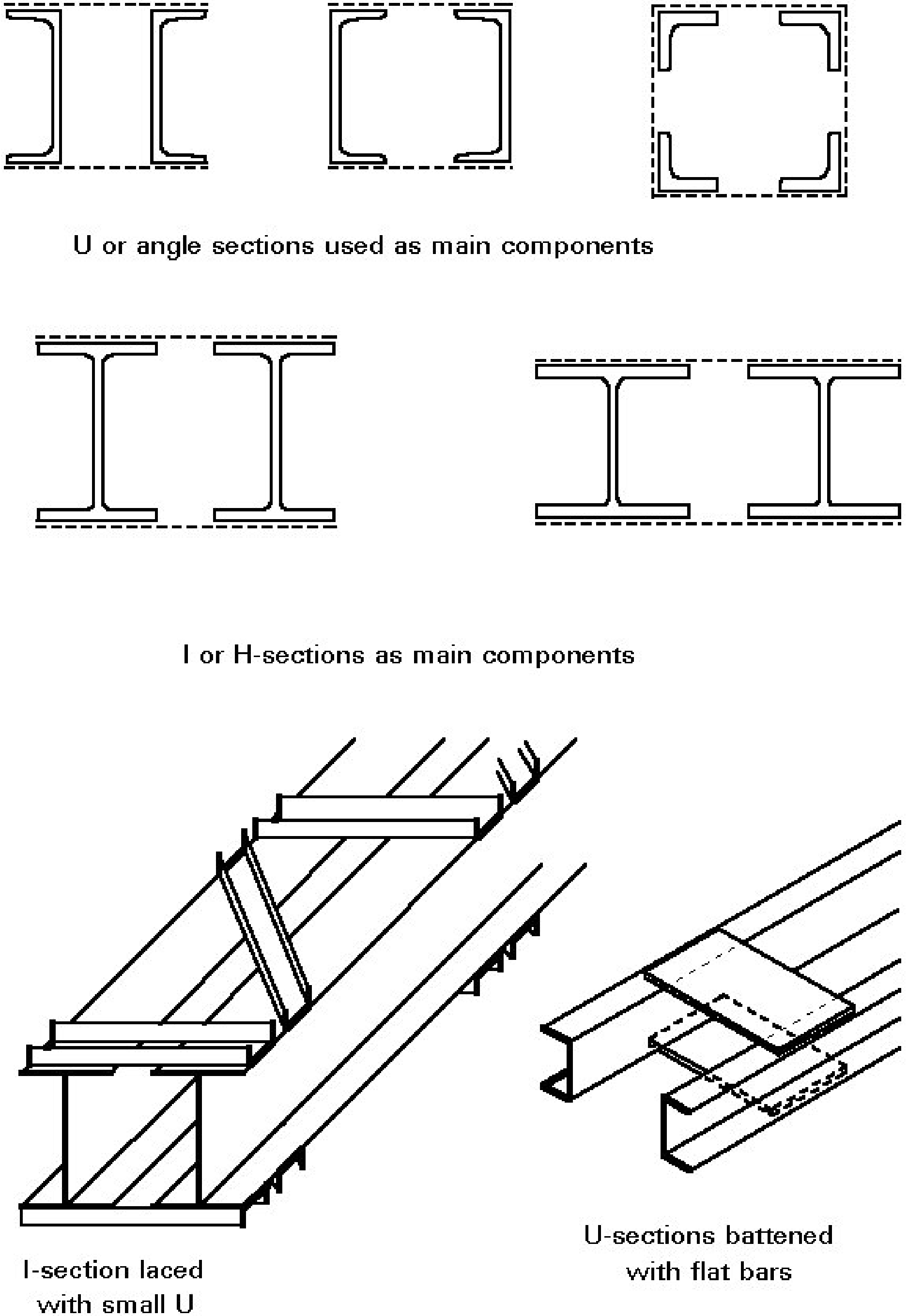


Figure 4 Built-up columns

Instability and bifurcation

- Stable, neutral and unstable equilibriums

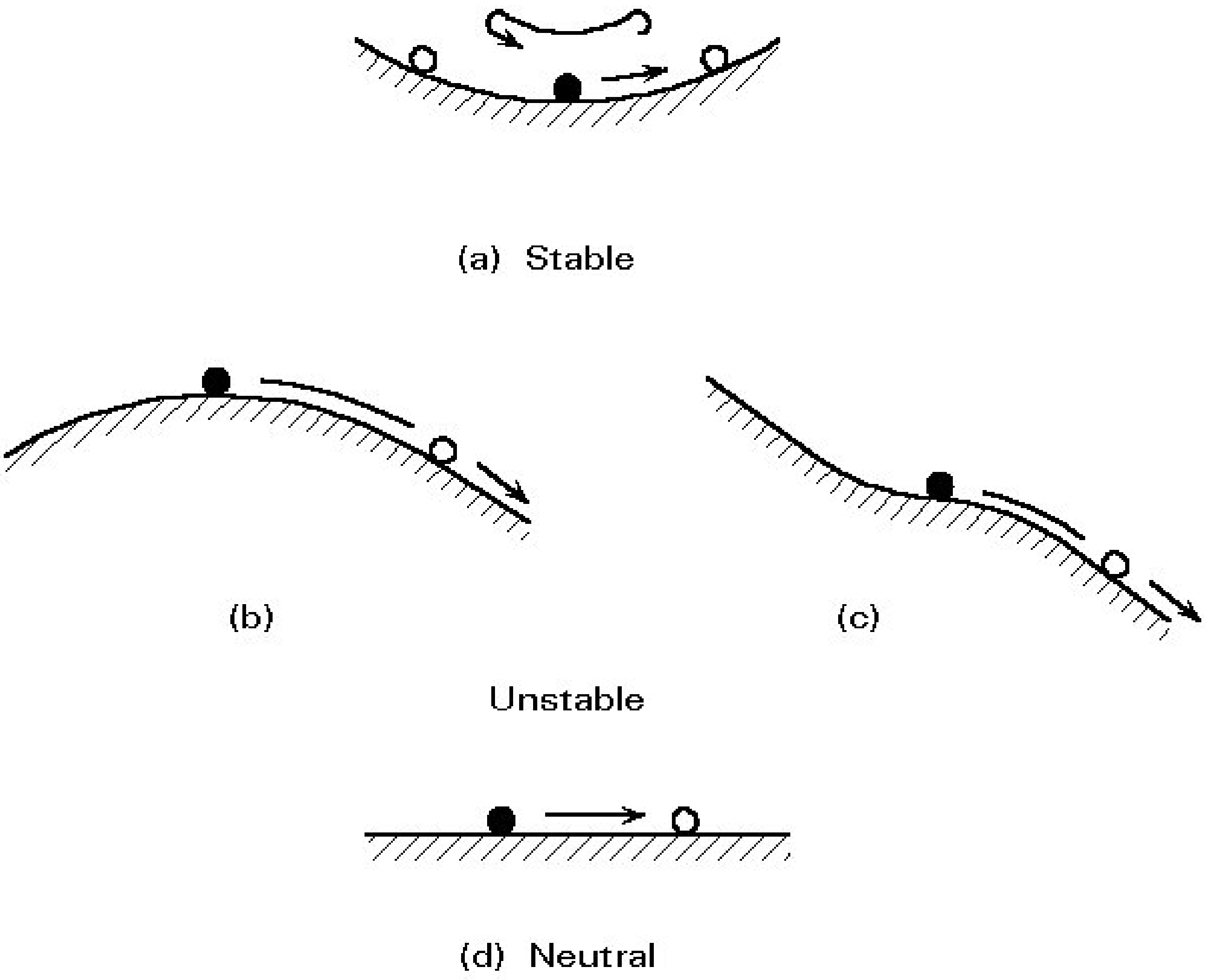
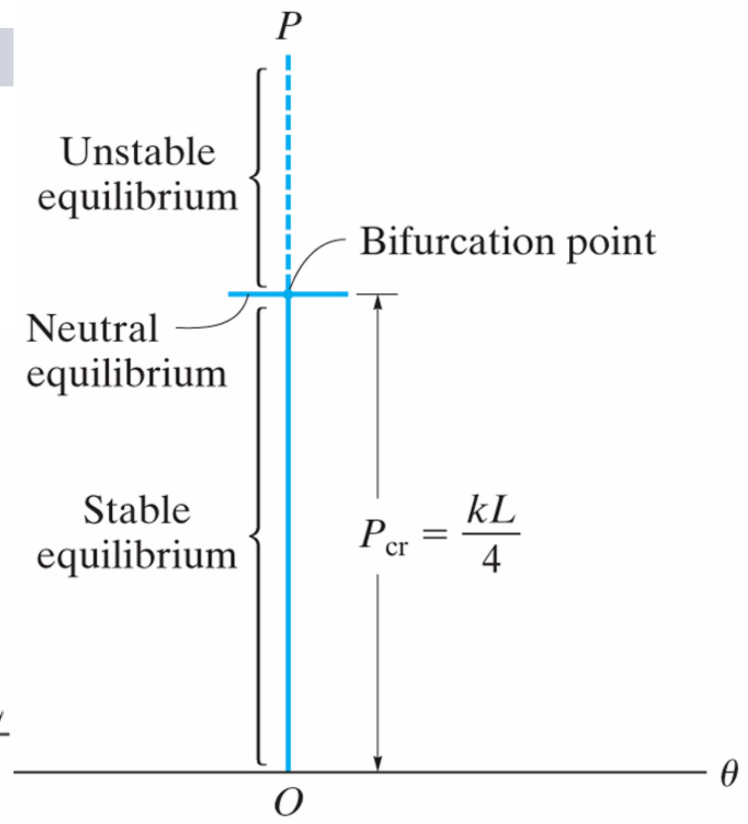
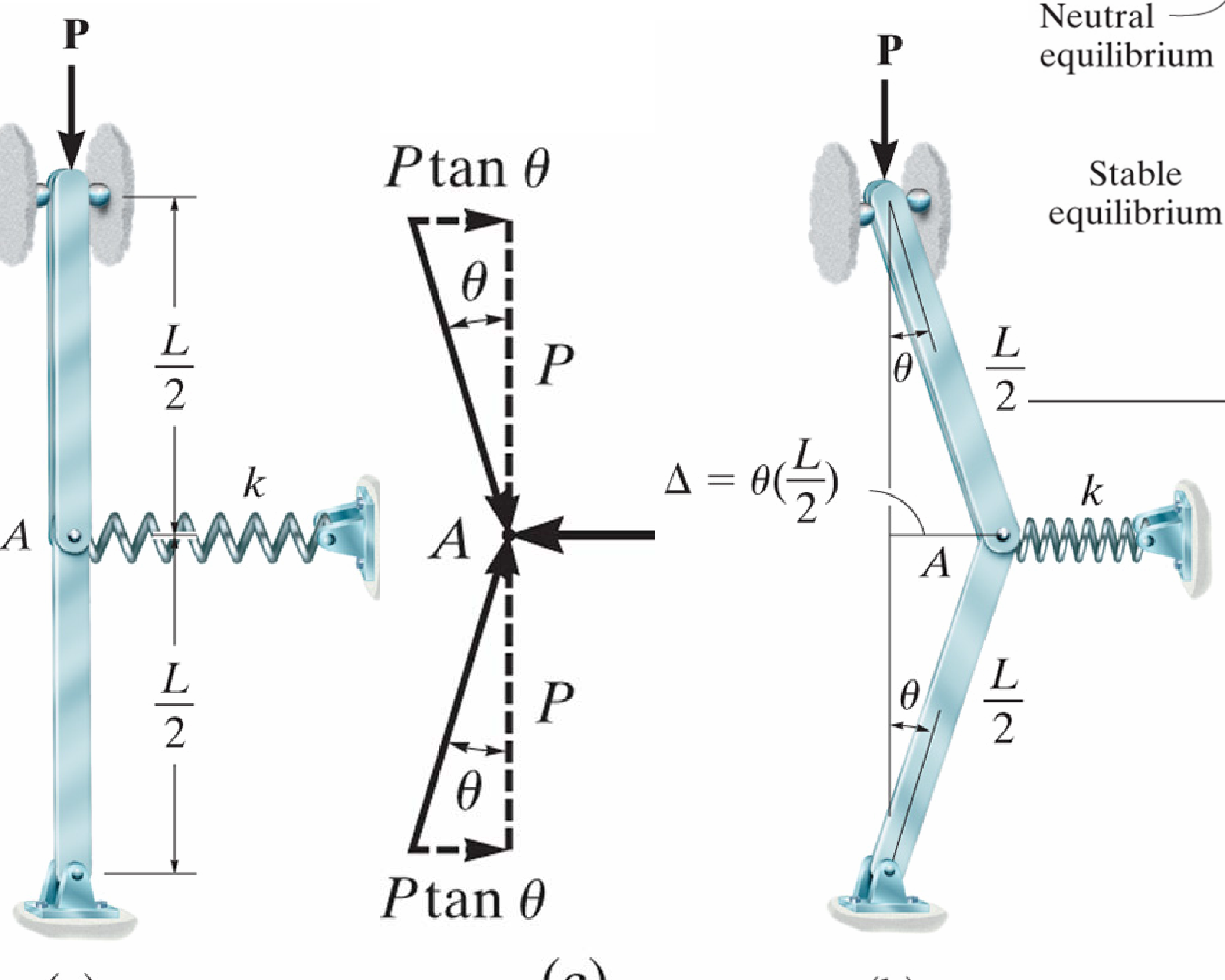


Figure 1 The three states of equilibrium

Buckling



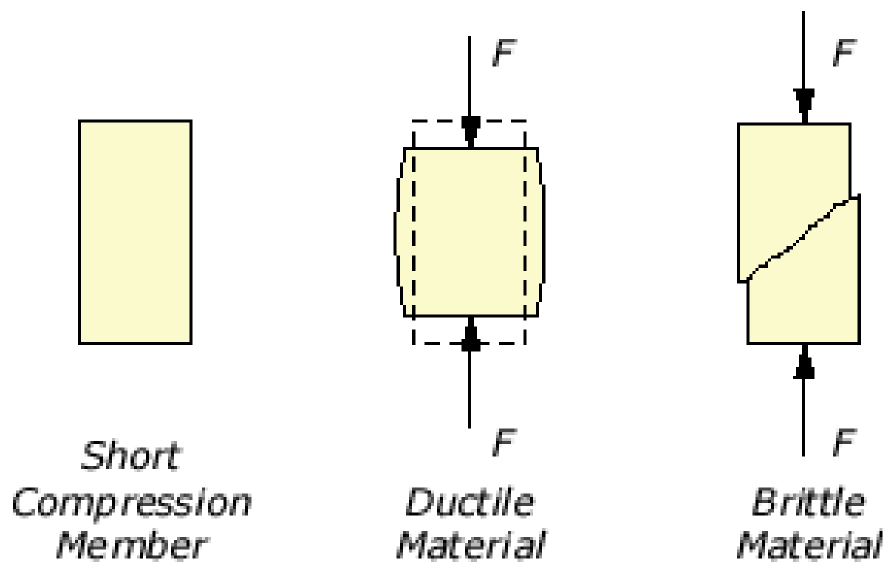
P
Unstable equilibrium
Bifurcation point
Neutral equilibrium
Stable equilibrium
$P_{cr} = \frac{kL}{4}$
O
θ



P
L/2
L/2
k
A
P tan θ
θ
P
A
P
θ
P tan θ
P
$\Delta = \theta(\frac{L}{2})$
θ
L/2
k
A
θ
L/2

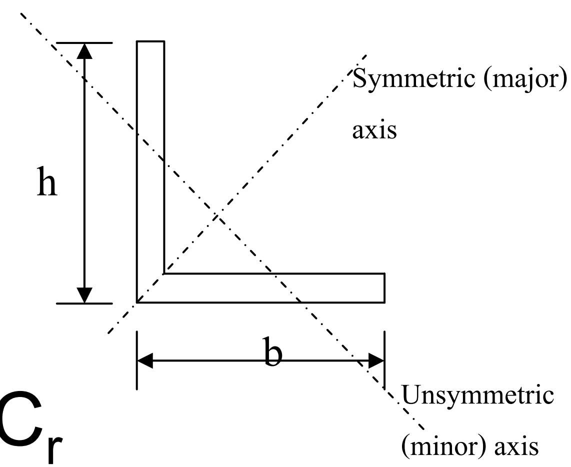
Instability and bifurcation

- Instability effect
  - To compress or not to compress?
  - Energy considerations



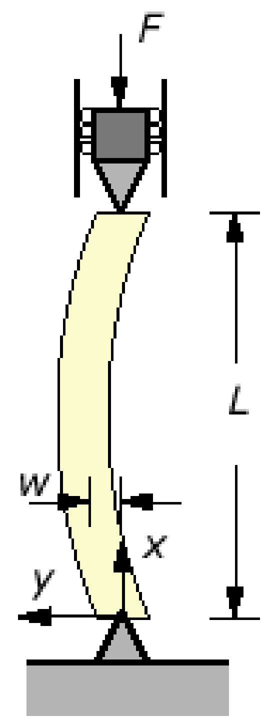
Compression terminology -review

- Moment of inertia $I_x = \int_A y^2 dA$
- Parallel axis theorem $I_{x'} = I_x + A\bar{x}^2$
- Radius of gyration $r = \sqrt{\frac{I}{A}}$
- Effective length kL
- Slenderness ratio kL/r
- Principal axes (major and minor)
- Critical Load P_{cr}
- Factored compressive strength, C_r



Compression members

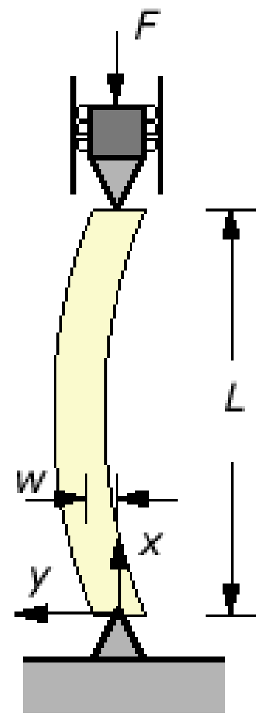
- Bucking
  - Elastic (Euler) buckling
  - Inelastic buckling
- Buckling modes
  - Overall buckling
    - Flexural buckling
    - Torsional buckling
    - Torsional-flexural buckling
  - Local buckling



Simply supported column subjected to axial load F

Elastic Buckling

- Equilibrium equation
  - Internal moment + applied moment = 0



Simply supported column subjected to axial load F

$$EI\frac{d^2w}{dx^2}+Pw=0; \qquad w=0\ @\ y=0; \qquad w=0\ @\ y=L$$

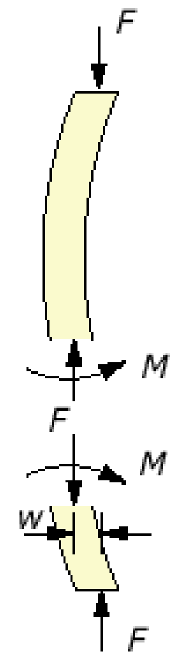
$$Solution: \qquad w=A\sin\frac{\pi x}{L} \quad satisfies\ the\ b.c.$$

Substituting into the differential equation,

$$EI\left(-A\left(\frac{\pi}{L}\right)^2\sin\frac{\pi x}{L}\right)+P\left(A\sin\frac{\pi x}{L}\right)=0$$

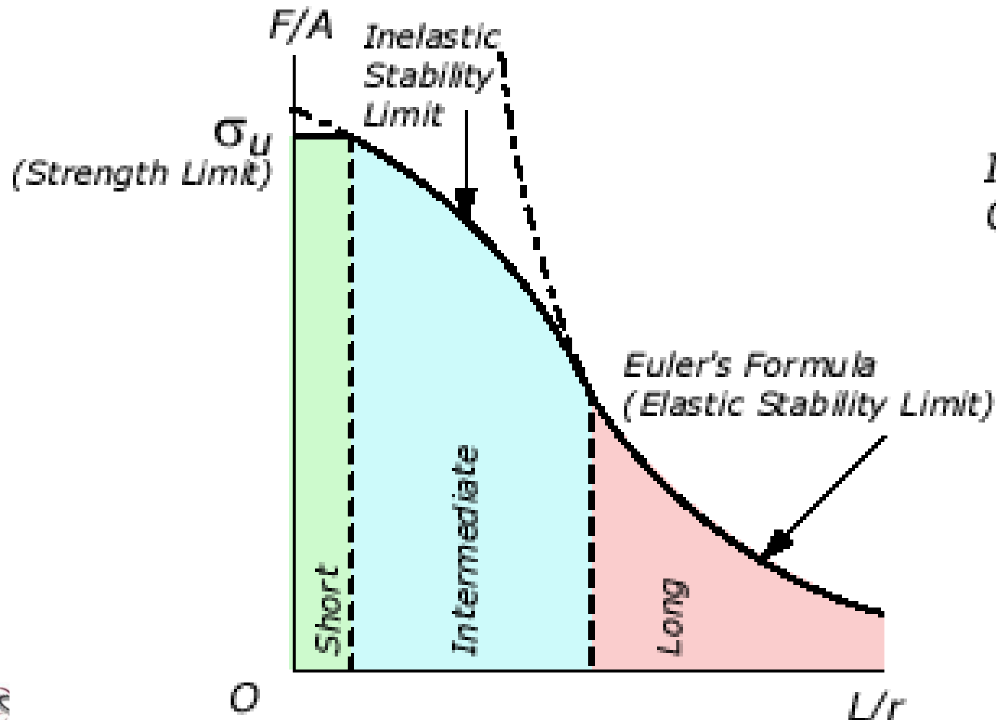
$$-\left(\frac{\pi}{L}\right)^2 EI+P=0$$

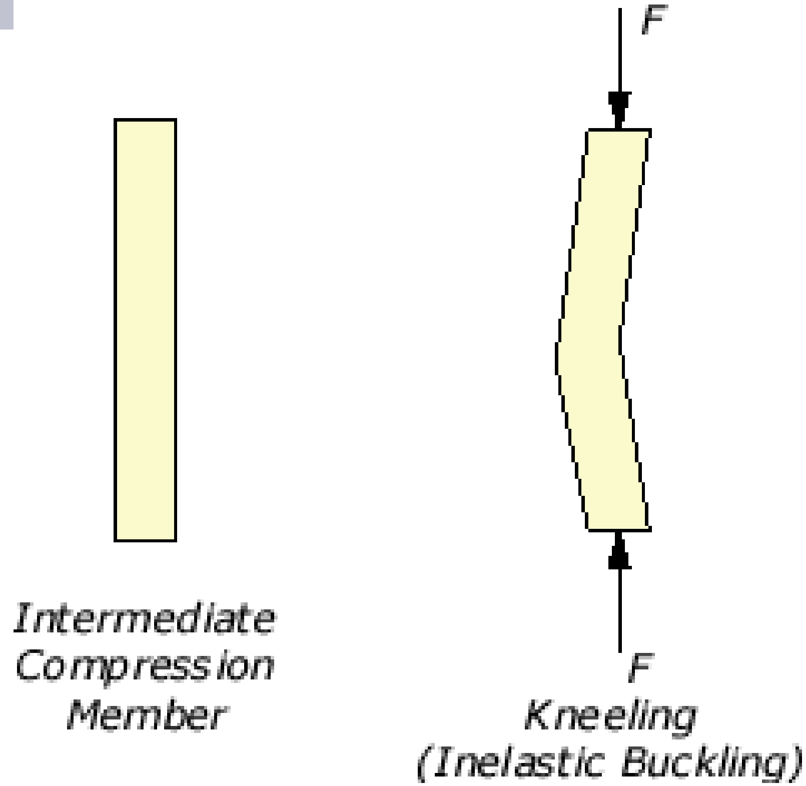
$$P_{cr}=\frac{\pi^2 EI}{L^2}$$



Free body diagram

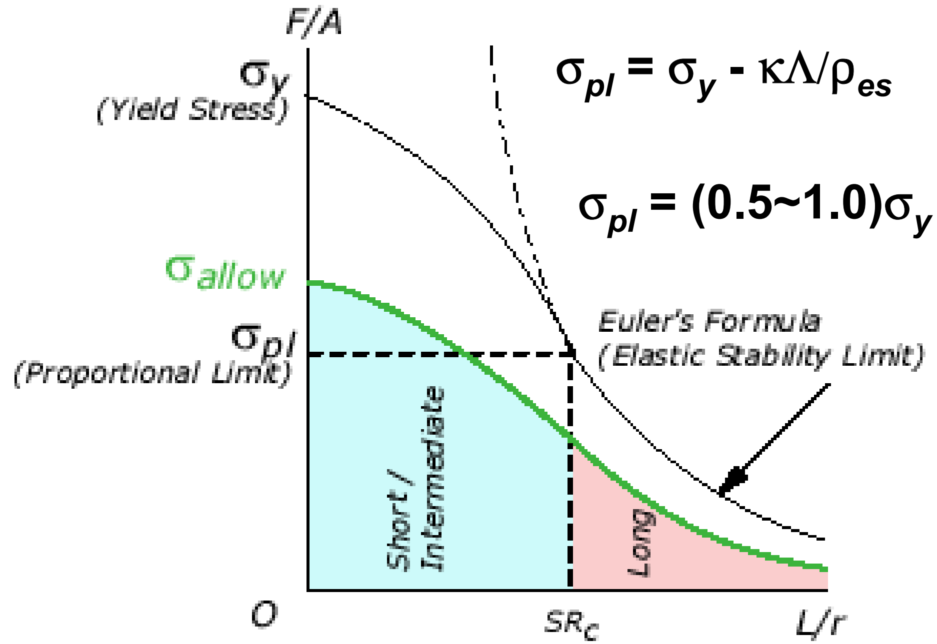
Inelastic Buckling

- 



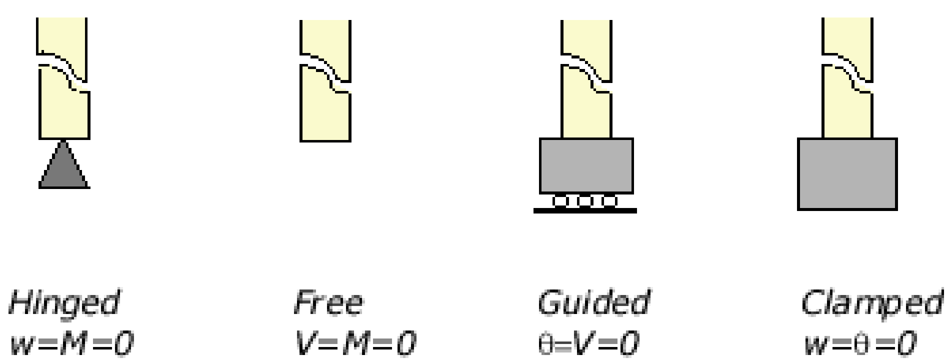
Compression members

- Moment of inertia
- Radius of gyration
- Effective length
- Slenderness ratio



$\sigma_{pl} = \sigma_y - \kappa\Lambda/\rho_{es}$

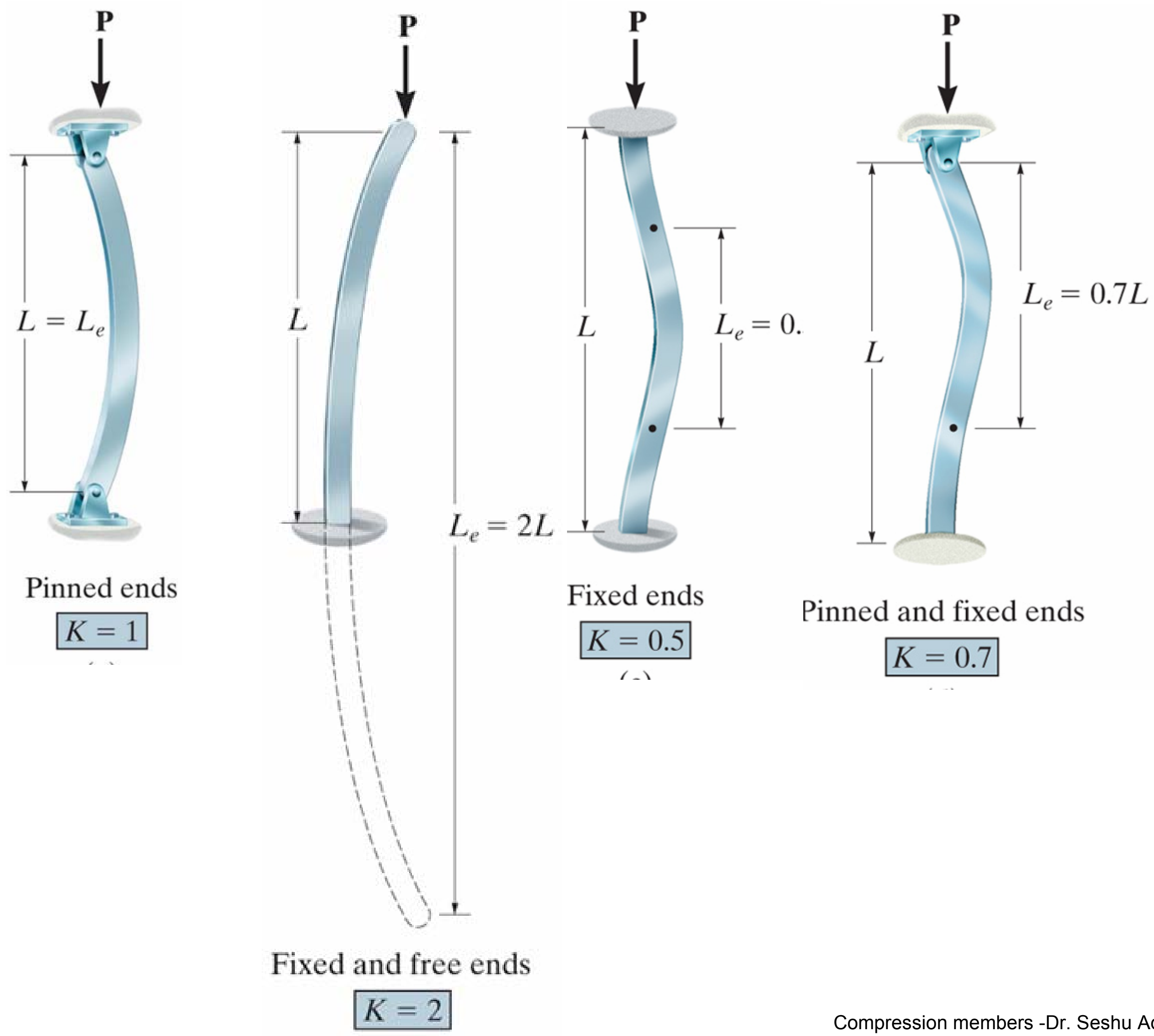
$\sigma_{pl} = (0.5\sim1.0)\sigma_y$



$$r = \sqrt{\frac{I}{A}}$$

Effective length factors

- Different end conditions give different lengths for equivalent half-sine wave



Theoretical Effective length factors

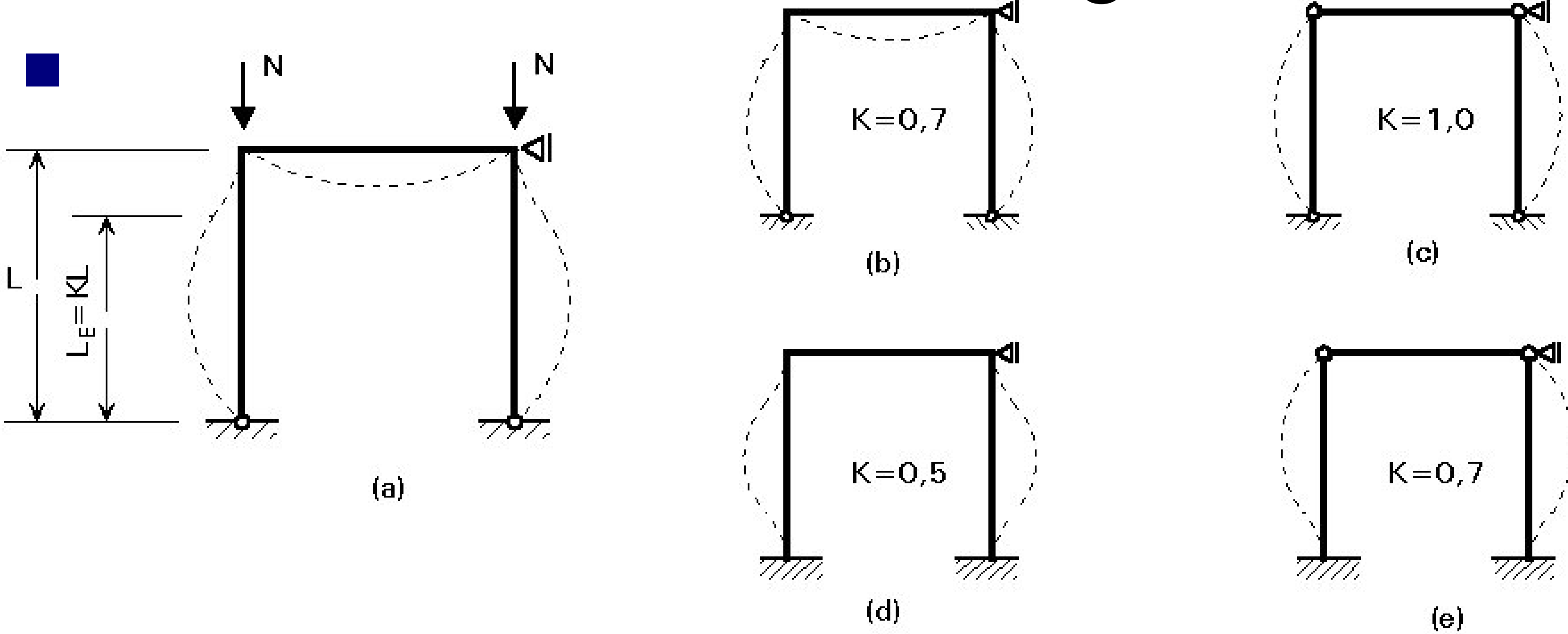


Figure 4 Buckling of a column in a non-sway frame

Theoretical Effective length factors

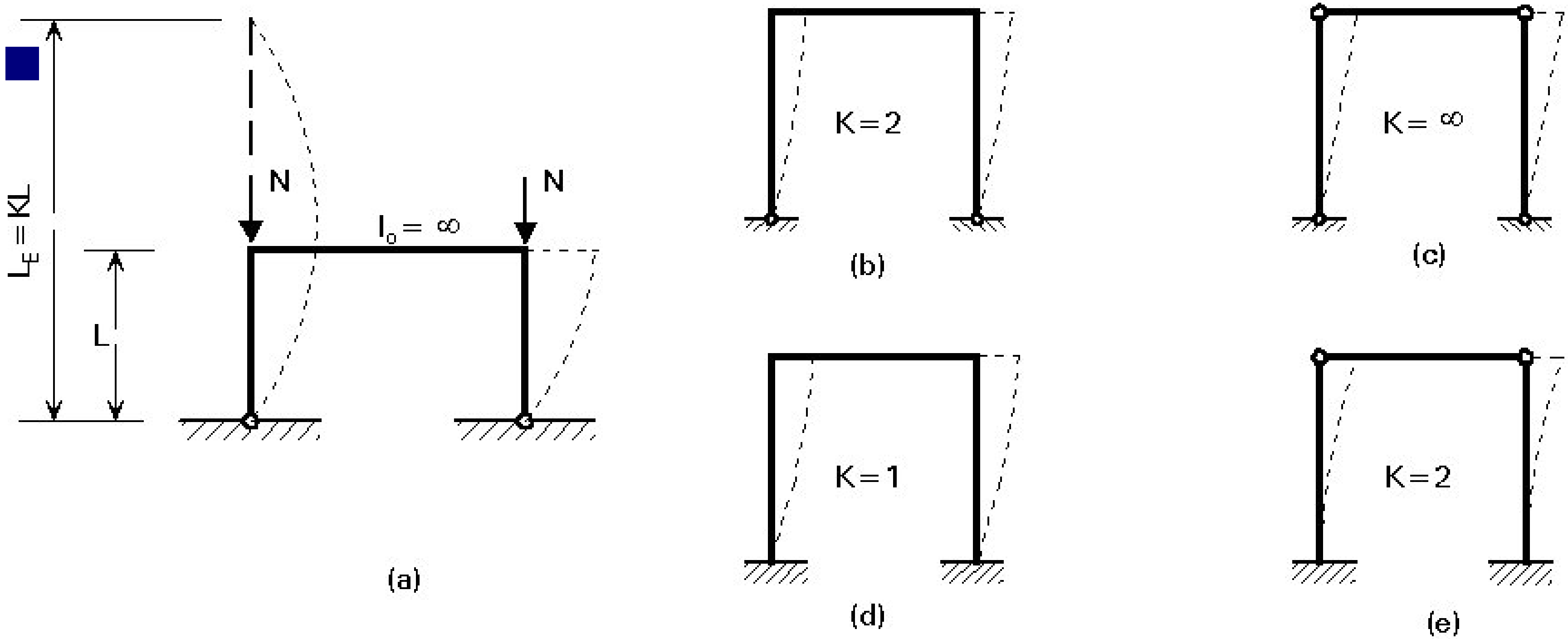
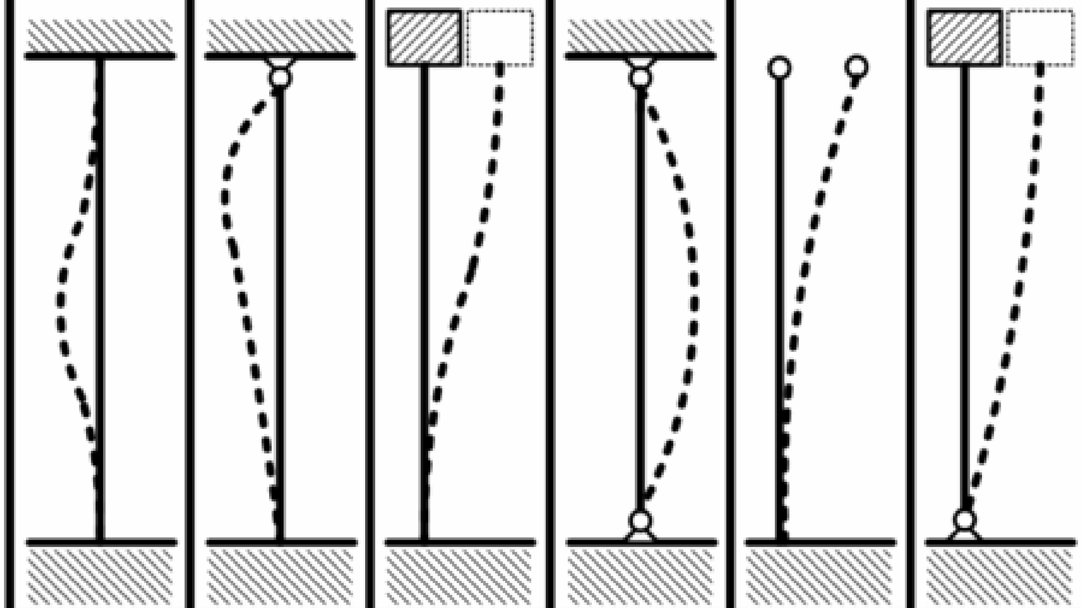
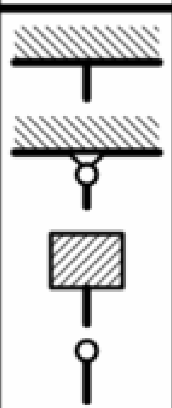


Figure 5 Buckling of a column in a sway frame

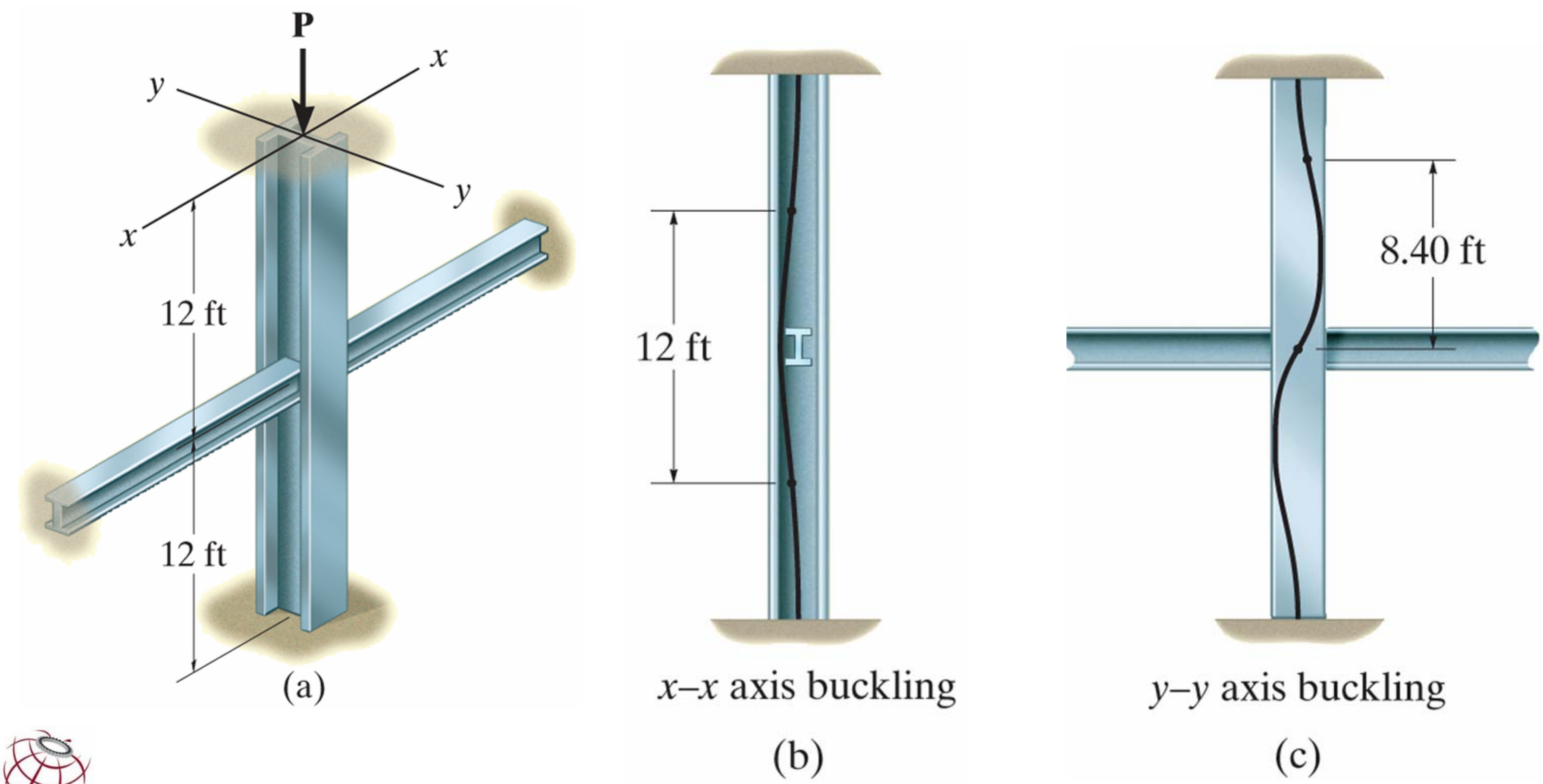
Effective length factors

- US practice

Buckled shape of column shown by dashed line						
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value K	0.65	0.80	1.2	1.0	2.10	2.0

End condition key		
		Rotation fixed and translation fixed
		Rotation free and translation fixed
		Rotation fixed and translation free
		Rotation free and translation free

Effective lengths in different directions



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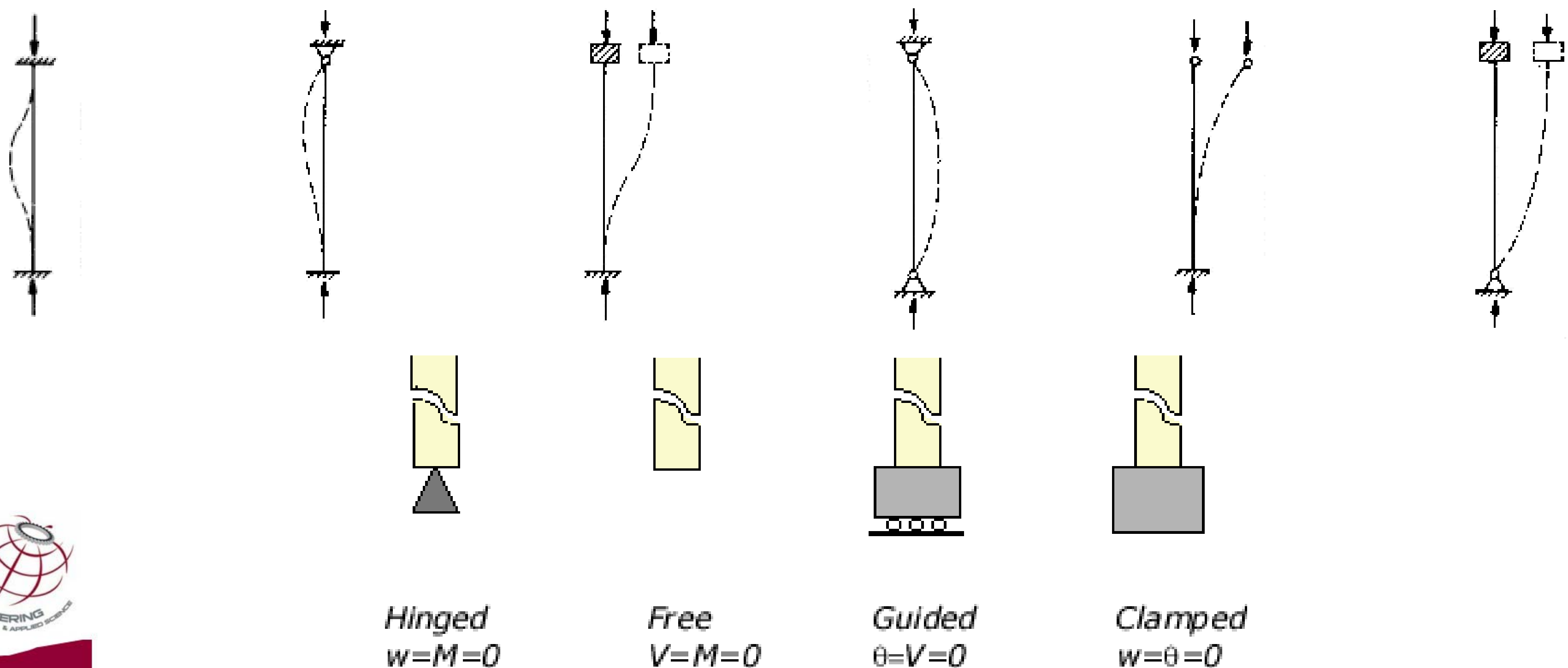
Effective length factors

Canadian practice

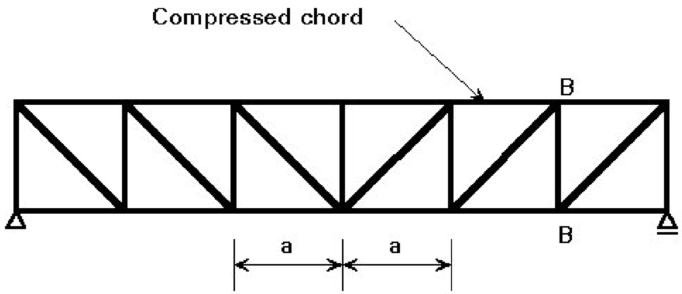
k = .65 **k = .8** **k = 1.2** **k = 1.0** **k = 2.0** **k = 2.0**

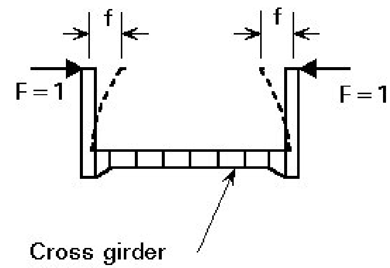


US recommended values Boundary Conditions	Theoretical Eff. Length, L_{eff}^{T}	Engrg. Eff. Length L_{eff}^{E}
Free-Free	L	($1.2 \cdot L$)
Hinged-Free	L	($1.2 \cdot L$)
Hinged-Hinged (Simply-Supported)	L	L
Guided-Free	$2 \cdot L$	($2.1 \cdot L$)
Guided-Hinged	$2 \cdot L$	$2 \cdot L$
Guided-Guided	L	$1.2 \cdot L$
Clamped-Free (Cantilever)	$2 \cdot L$	$2.1 \cdot L$
Clamped-Hinged	$0.7 \cdot L$	$0.8 \cdot L$
Clamped-Guided	L	$1.2 \cdot L$
Clamped-Clamped	$0.5 \cdot L$	$0.65 \cdot L$

Canadian recommended values – Appendix F CAN/CSA/S16-01 **Boundary Conditions**	**Theoretical Eff. Length, L_{eff}^T**	**Engrg. Eff. Length L_{eff}^E**
Free-Free	***L***	**(*1.2·L*)**
Hinged-Free	***L***	**(*1.2·L*)**
Hinged-Hinged (Simply-Supported)	***L***	***L***
Guided-Free	***2·L***	**(*2.0·L*)**
Guided-Hinged	***2·L***	***2·L***
Guided-Guided	***L***	***1.2·L***
Clamped-Free (Cantilever)	***2·L***	***2.0·L***
Clamped-Hinged	***0.7·L***	***0.8·L***
Clamped-Guided	***L***	***1.2·L***
Clamped-Clamped	***0.5·L***	***0.65·L***

Effective lengths in frame columns





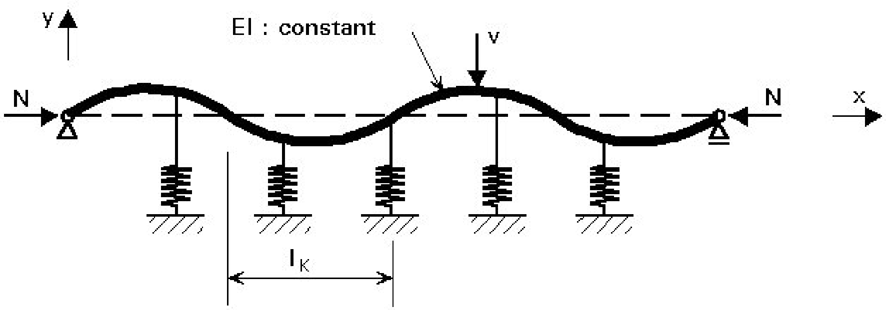


Figure 7 Buckling of a bar with elastic supports

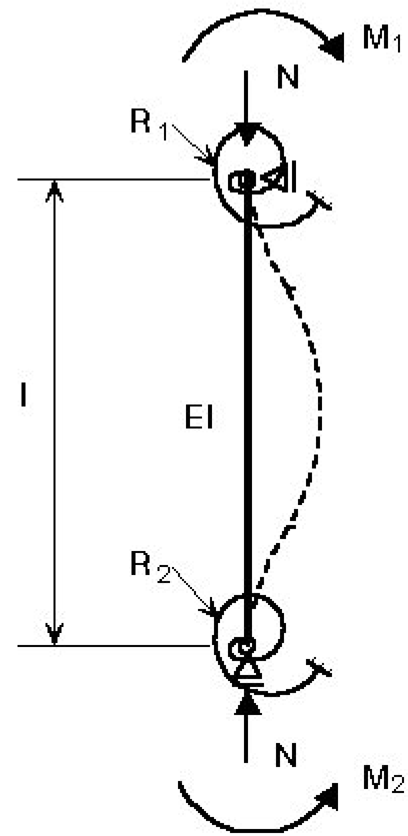


Figure 6 Subassemblage for Donnell's formula

Effective lengths in frame columns

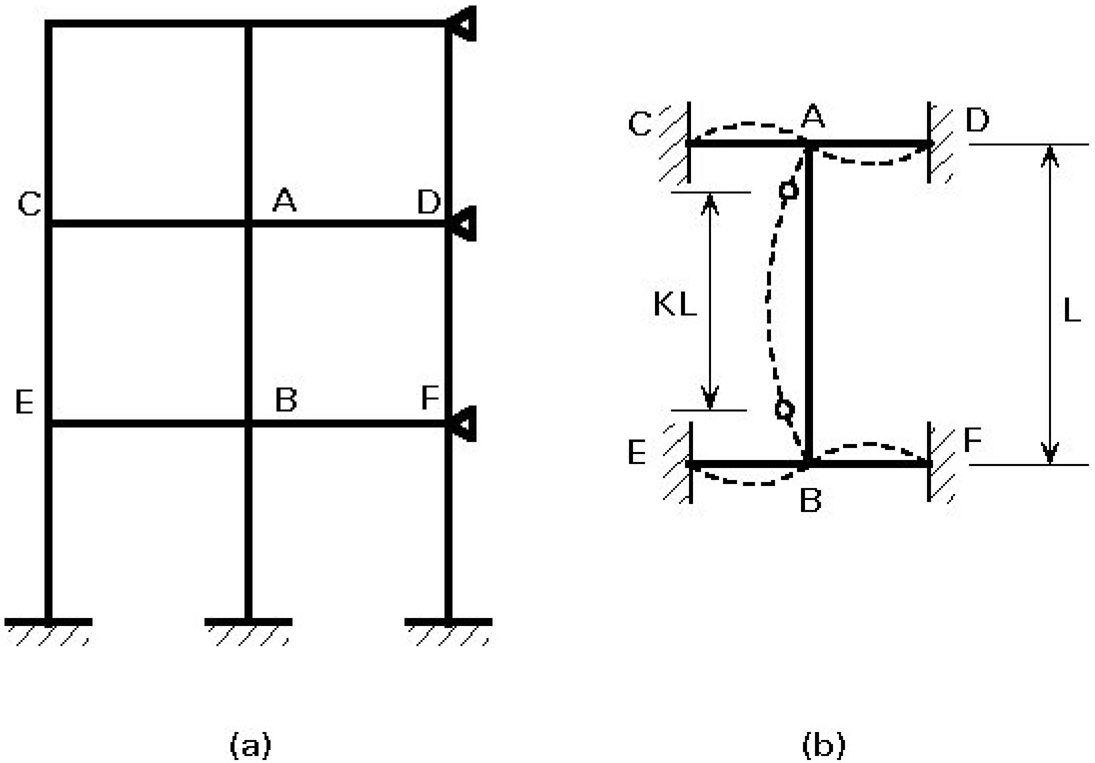


Figure 8 Example of substitute frame

Real columns -Factors for consideration

- Partially plastic buckling
- Initial out-of-straightness (L/2000 to L/1000)

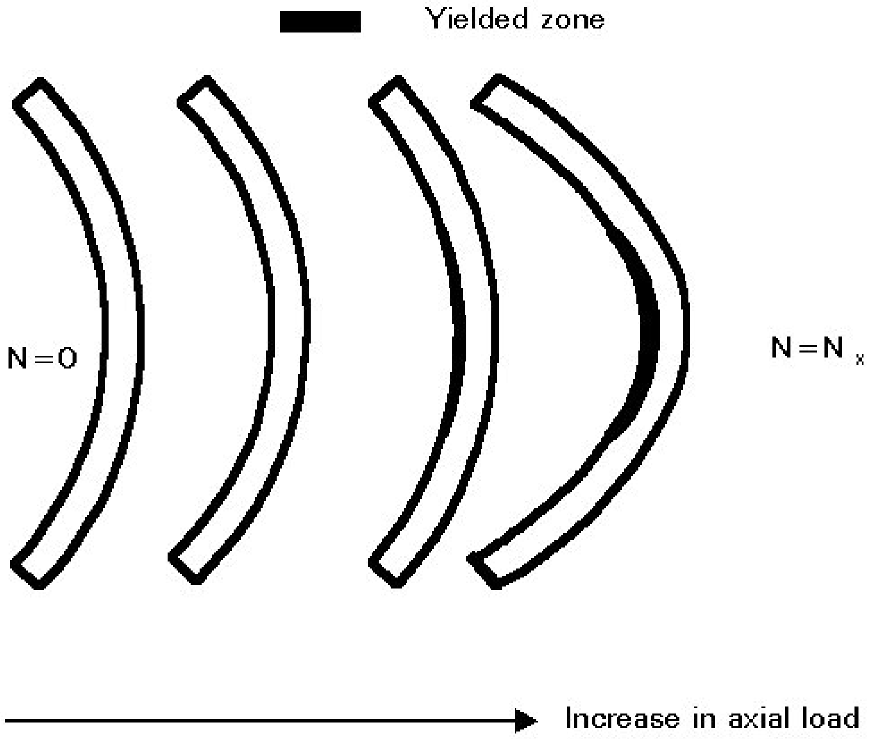
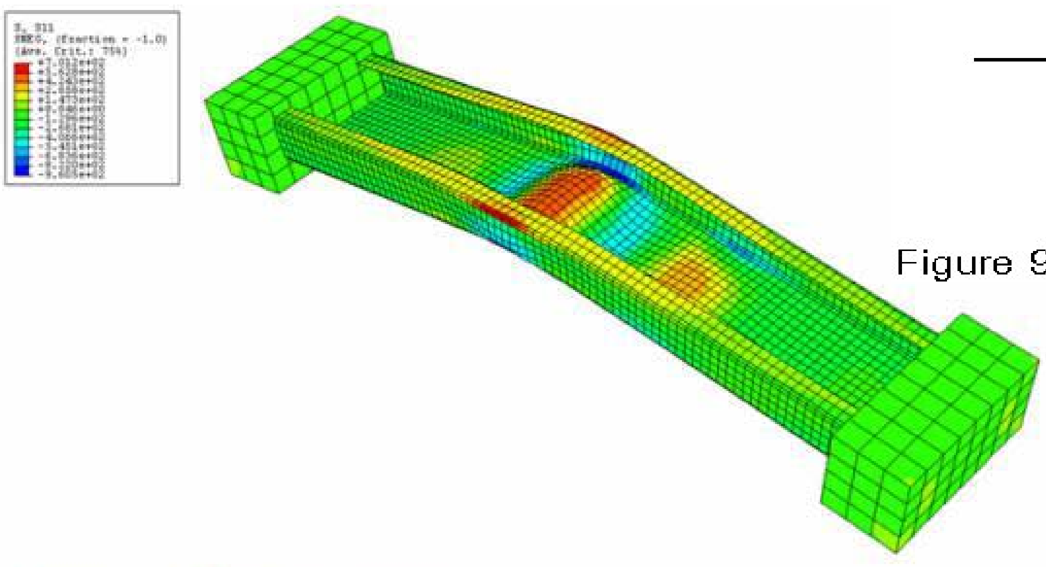
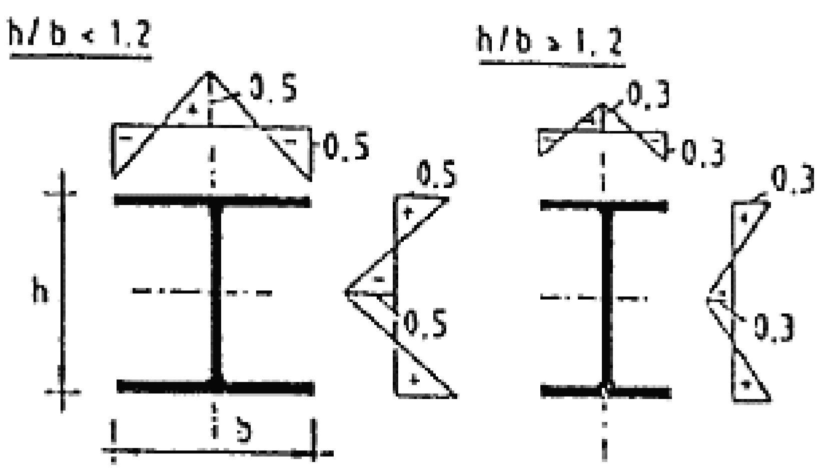


Figure 9 Spread of yielding as collapse approaches



Real columns - Factors for consideration

- Residual stresses in Hot-rolled shapes (idealized)



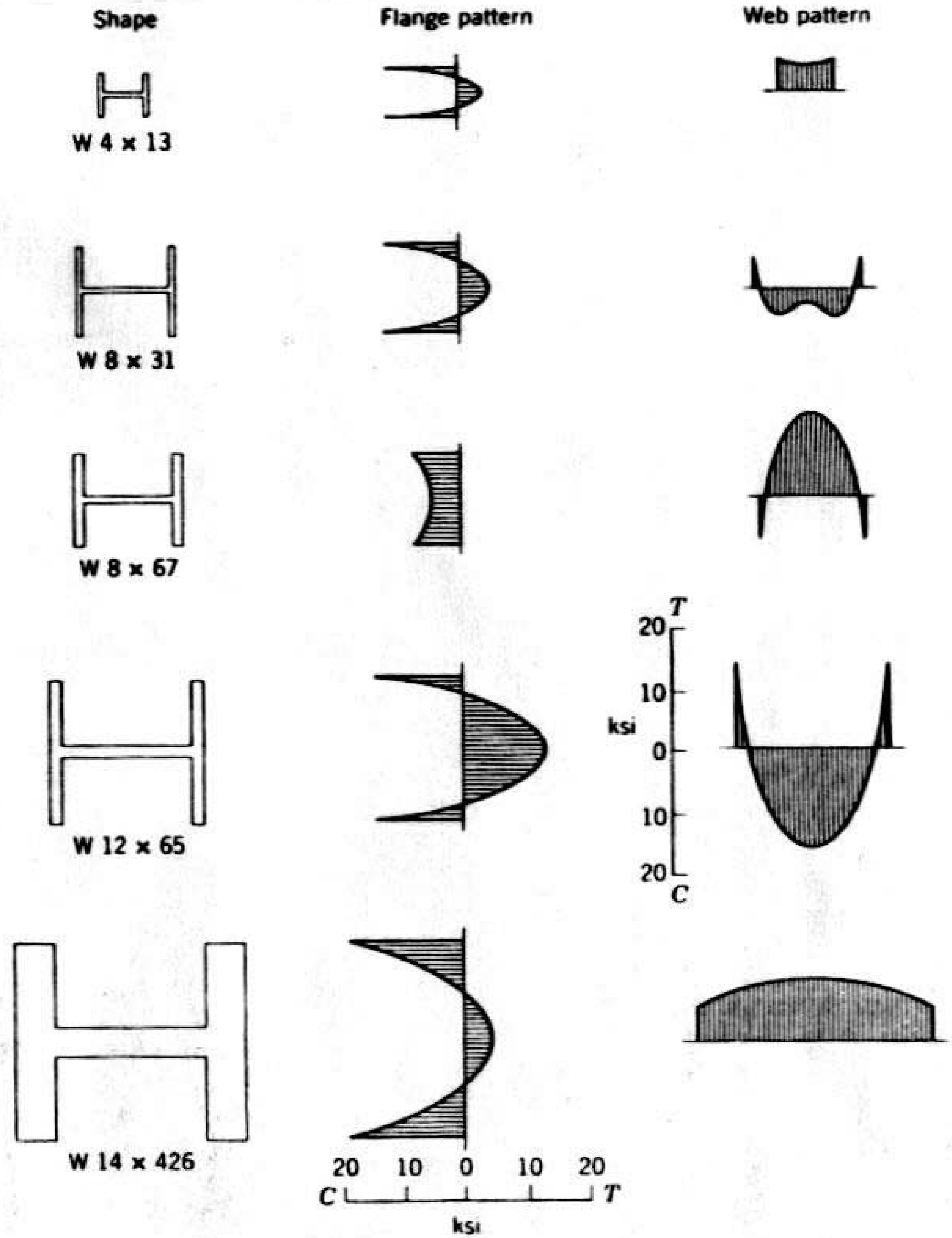


Fig. 3.3 Residual-stress distribution in rolled wide-flange shapes.

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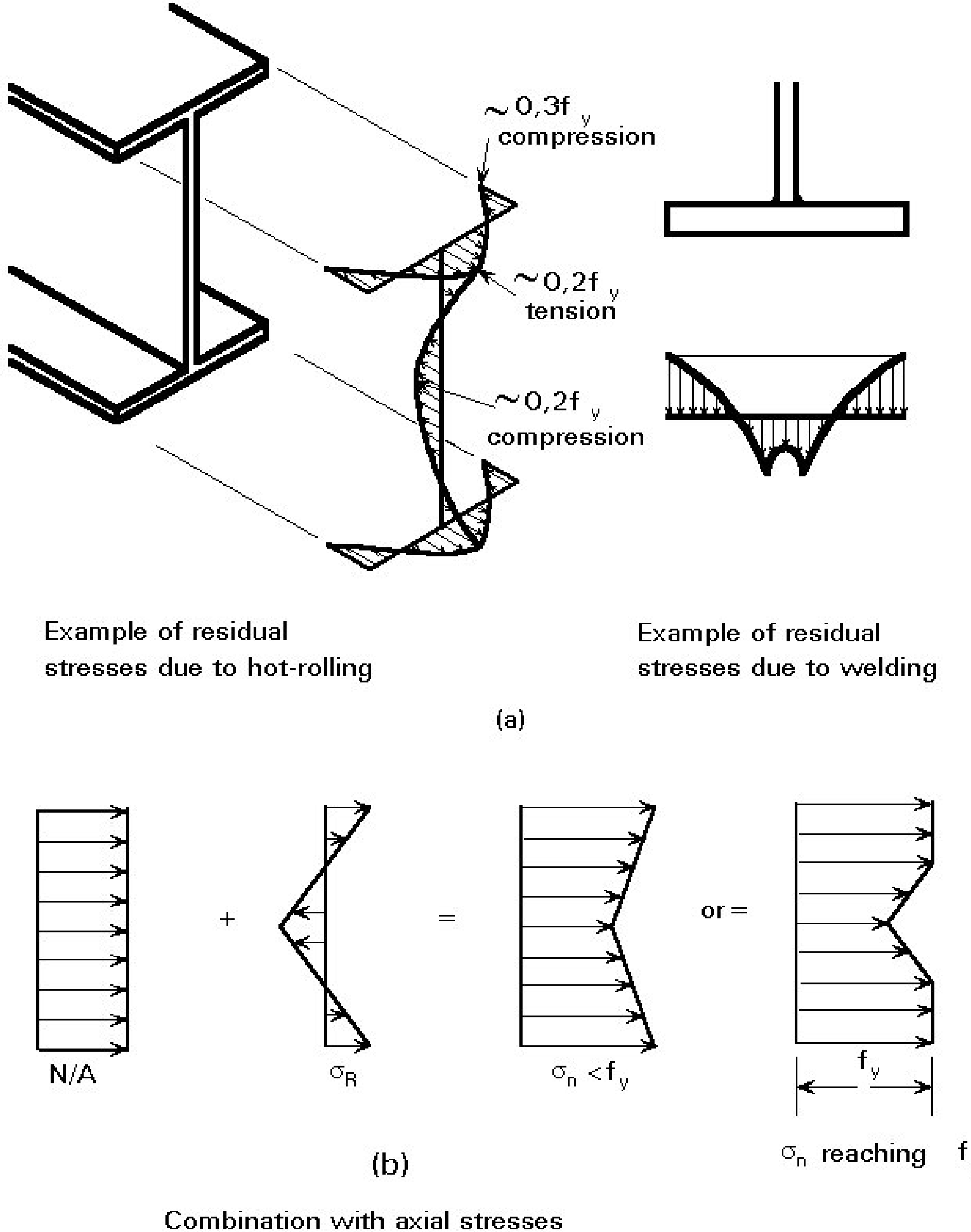
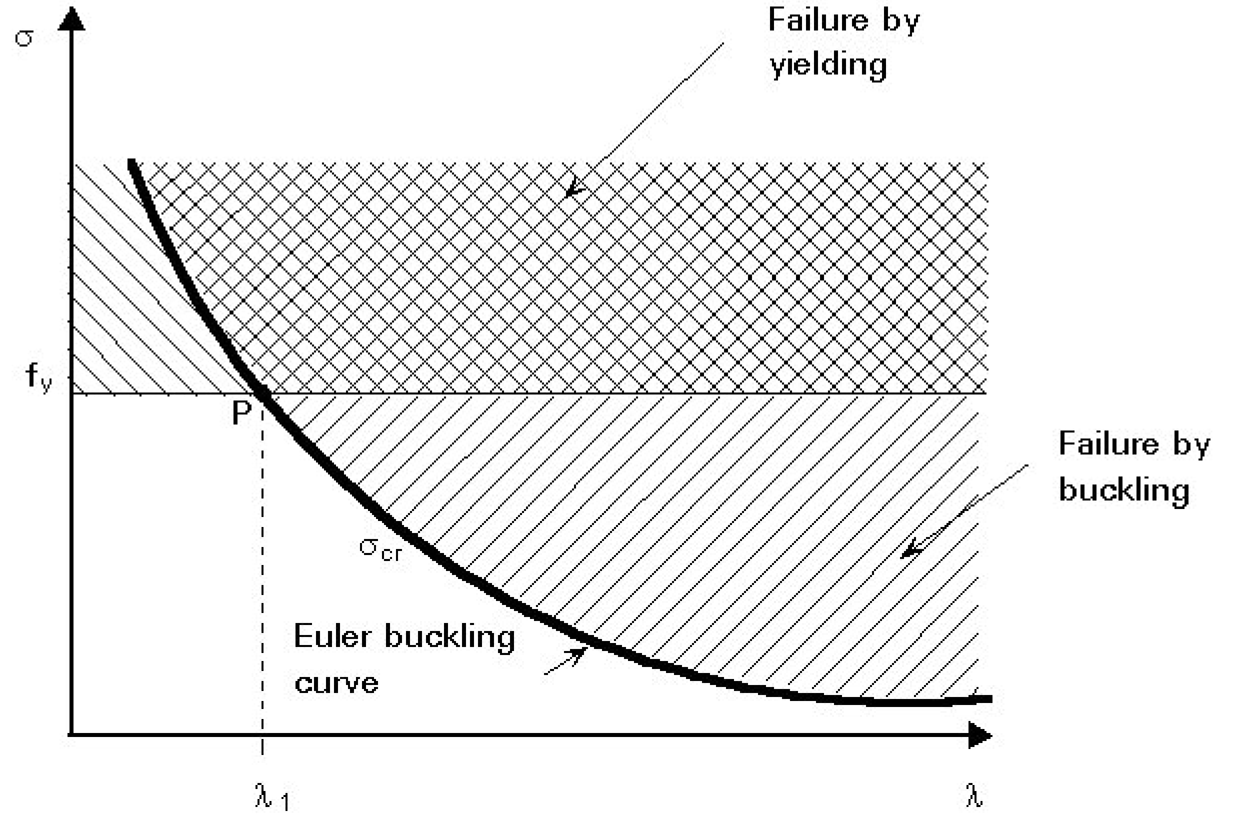
Real columns - Factors for consideration

- Residual stresses in Hot-rolled shapes (idealized)



Perfect column failure

-



Perfect column failure

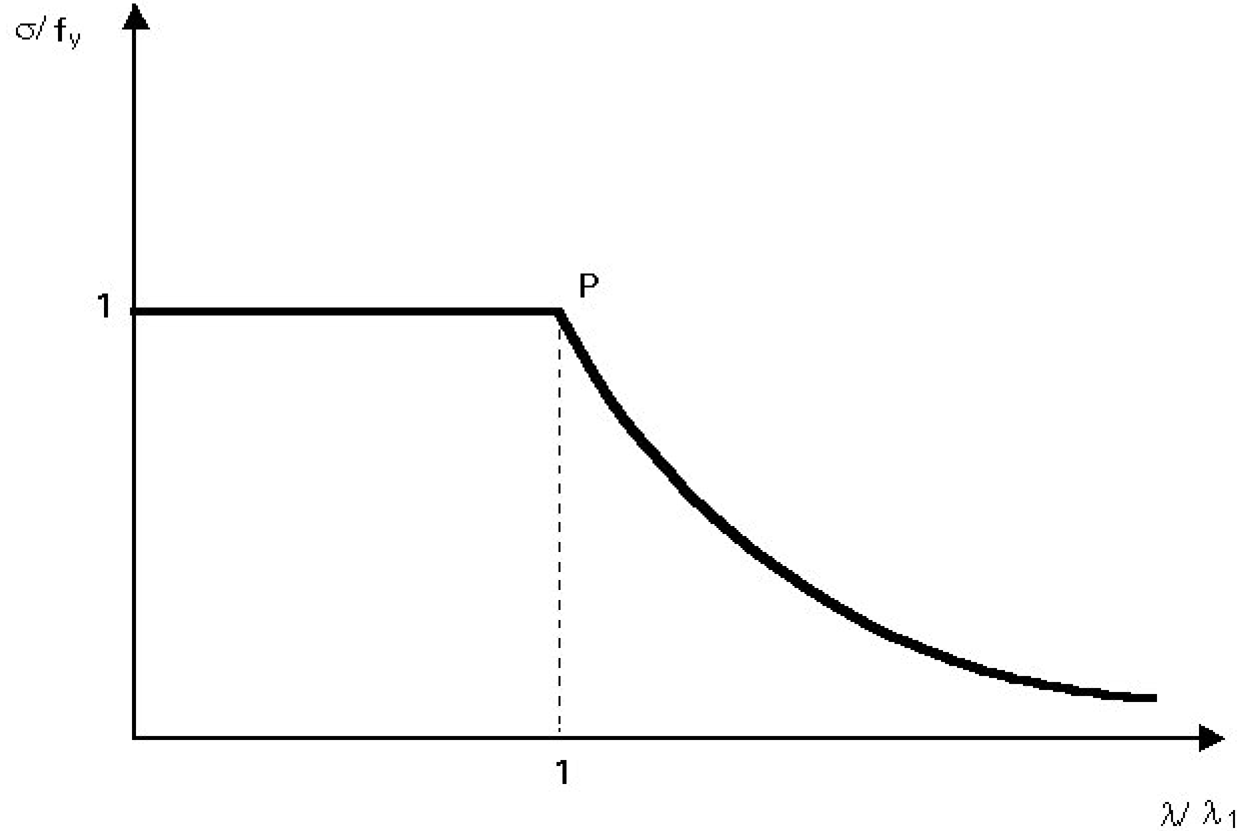


Figure 10 Non-dimensional buckling curve

Practical column failure

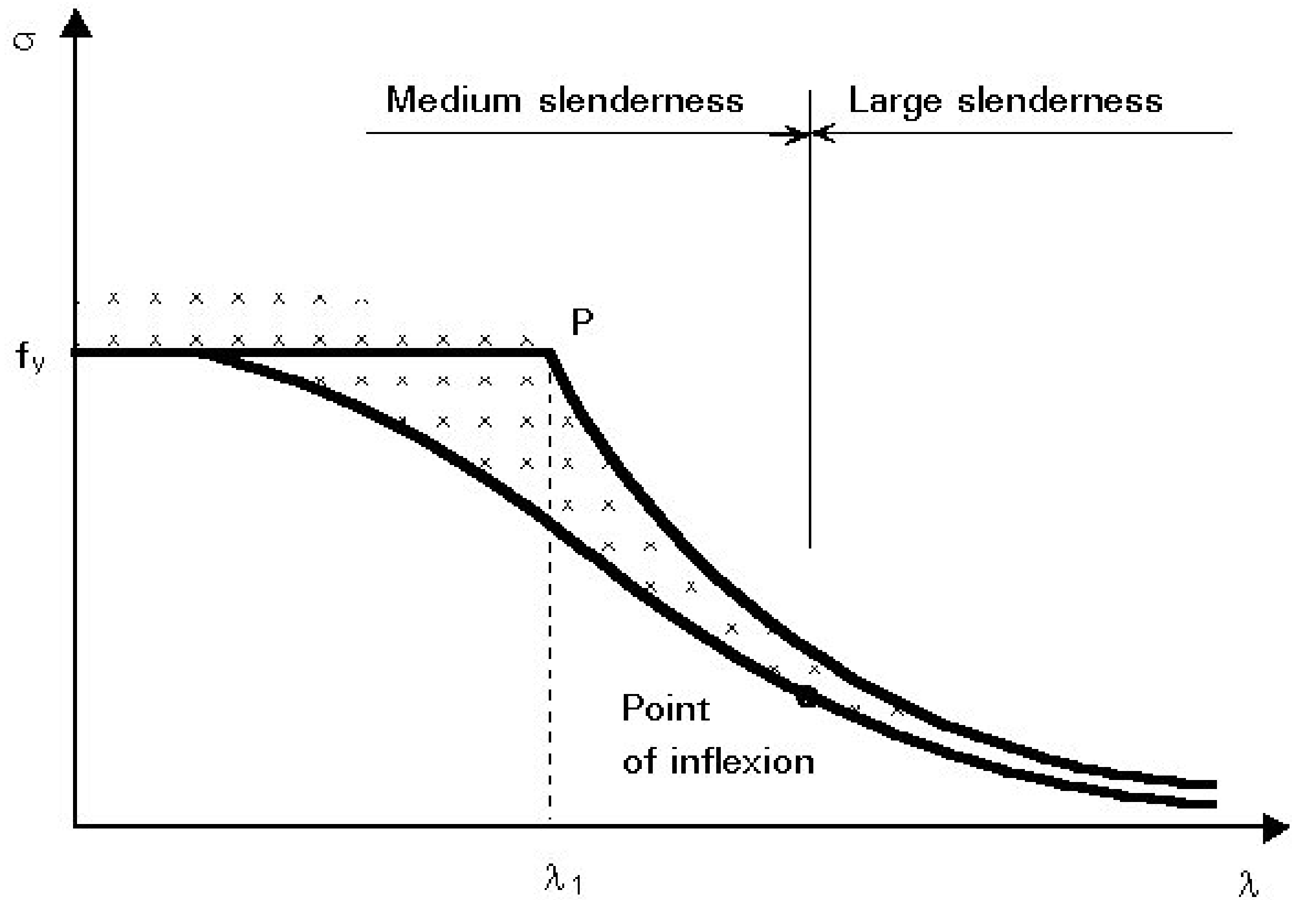


Figure 11 Real column test results
and buckling curves

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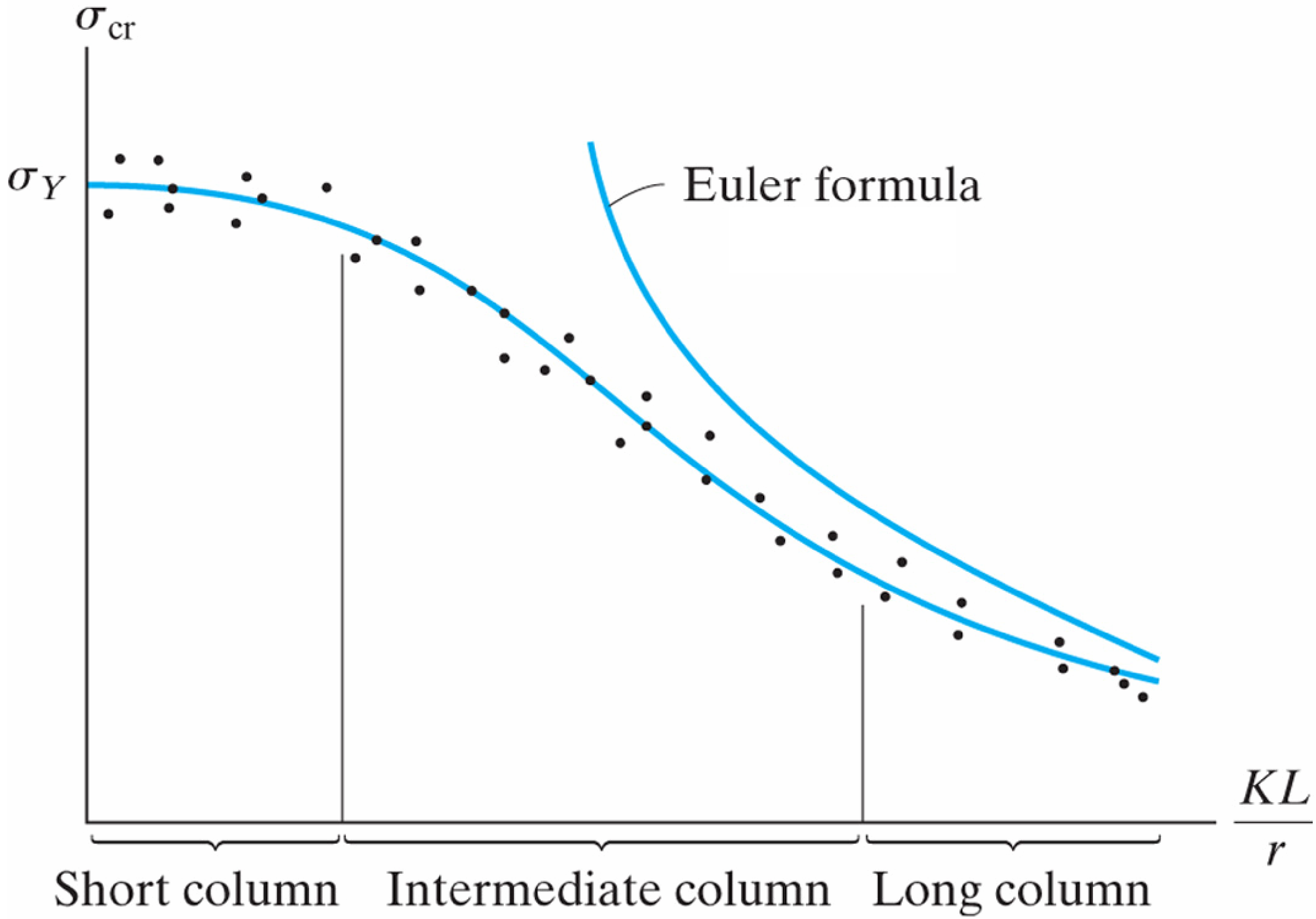
Column curve



Material	Short Column (Strength Limit)	Intermediate Column (Inelastic Stability Limit)	Long Column (Elastic Stability Limit)
	Slenderness Ratio ($kL/r = L_{eff} / r$)		
Structural Steel	$kL/r < 40$	$40 < kL/r < 150$	$kL/r > 150$
Aluminum Alloy AA 6061 - T6	$kL/r < 9.5$	$9.5 < kL/r < 66$	$kL/r > 66$
Aluminum Alloy AA 2014 - T6	$kL/r < 12$	$12 < kL/r < 55$	$kL/r > 55$
Wood	$kL/r < 11$	$11 < kL/r < (18\sim30)$	$(18\sim30)<kL/r<50$

Over-all buckling

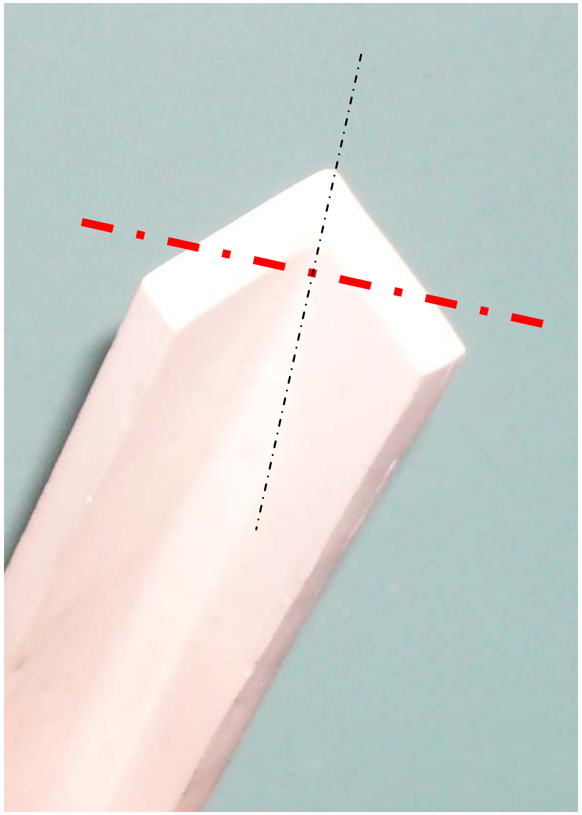
- Flexural
- Torsional
- Torsional-flexural

Flexural Buckling

- About minor axis (with higher kL/R) for doubly symmetric shapes
- About minor axis (the unsymmetric axis) for singly symmetric shapes

1964 Alaska quake, EqIIS collection

Flexural Buckling



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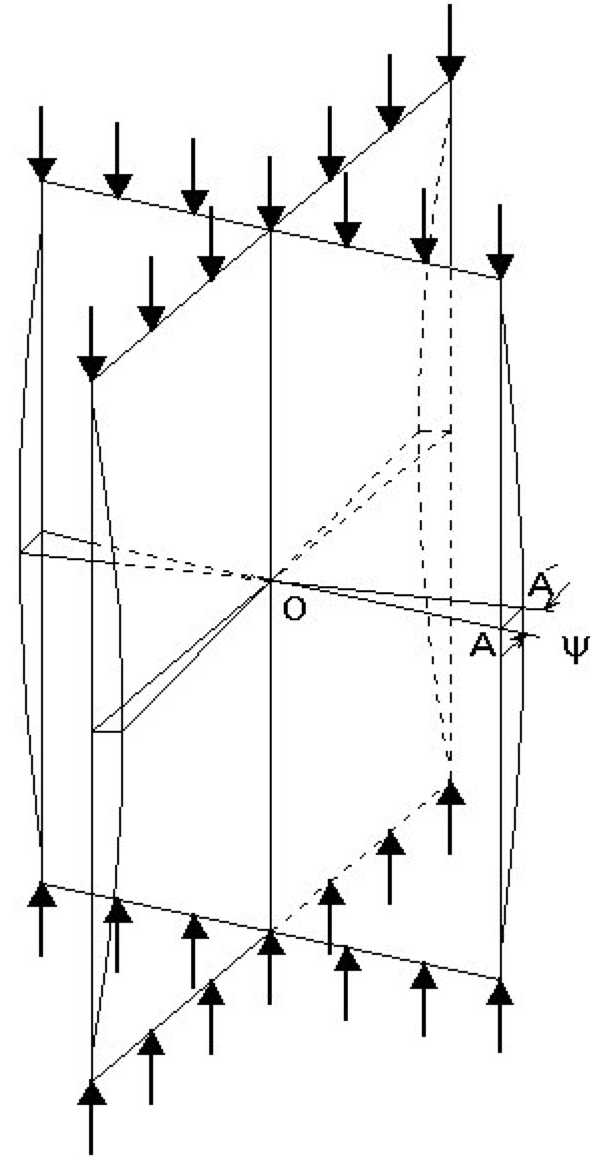
Torsional buckling

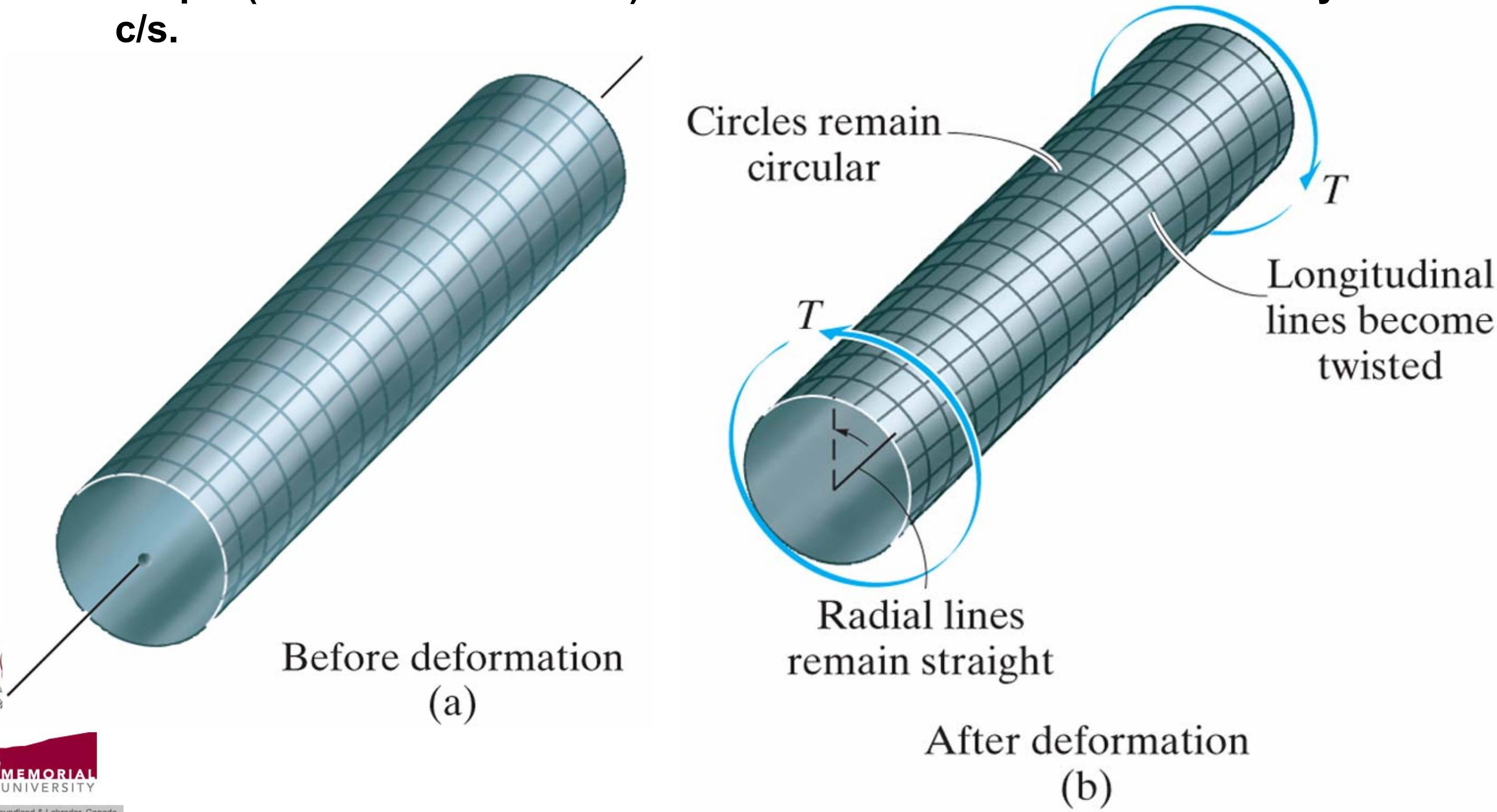
- Short lengths
  - Usually kL/r less than approx. 50
  - doubly symmetric sections
    - Wide flange sections, cruciform sections, double channels, point symmetric sections, ….
  - **Not for closed sections** such as HSS since they are very strong in torsion



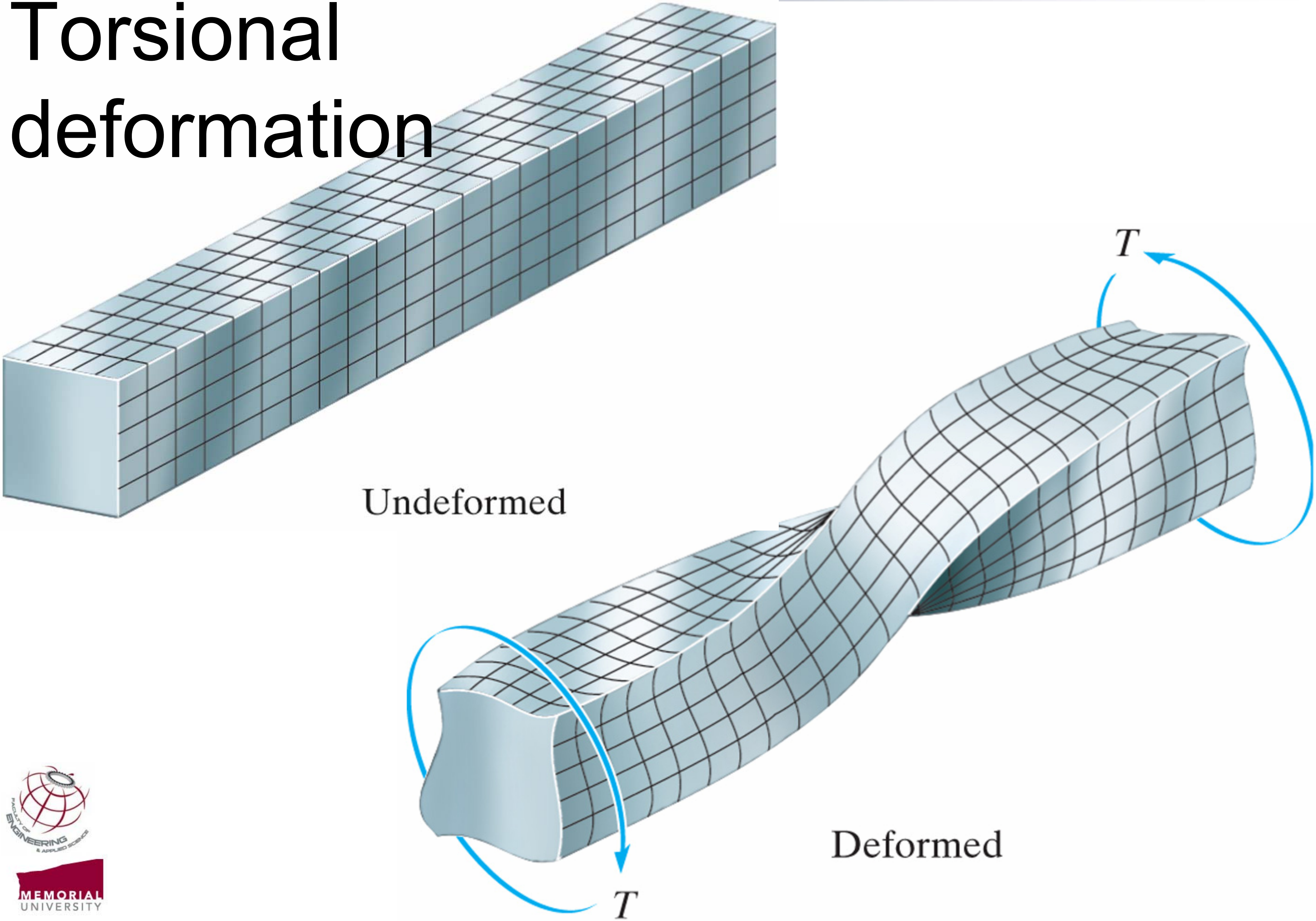
Torsional buckling of cruciform section

Torsion

Torque is a moment that causes twisting along the length of a bar. The twist is also the torsional deformation. For a circular shaft, the torque (or torsional moment) rotates each c/s relative to the nearby c/s.



Torsional deformation



Torsion of non-circular sections

- Torsion of non-circular sections involves torsional shear and warping.
- Torsional shear needs the use of torsion constant J.
  - J is similar to the use of polar moment of inertia for circular shafts.
  - $J=\Sigma bt^3/3$
- Warping calculation needs the use od the constant C_w.
- Both J and C_w are listed in the Handbook
- In addition, we need to use the effective length in torsion (k_zL_z). Usually, k_z is taken as 1.0

Torsional buckling of open sections

- Buckling in pure torsional mode (not needed for HSS or closed sections):
  - K_z is normally taken as 1.0.
  - C_w, J, r_x, r_y are given in the properties tables, x and y are the axes of symmetry of the section.
  - E= 200 000 MPa (assumed), G=77 000 MPa (assumed).

$$F_{ez} = \frac{1}{A\bar{r}_o^2}\left(\frac{\pi^2 E C_w}{(K_z L)^2} + GJ\right)$$

$$\bar{r}_o^2 = x_o^2 + y_o^2 + r_x^2 + r_y^2$$

$$\lambda = \sqrt{\frac{F_y}{F_e}}$$

$$C_r = \phi A F_y \left(1 + \lambda^{2n}\right)^{-1/n}$$

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Shear centre

- Sections always rotate about shear centre
- Shear centre lies on the axis of symmetry

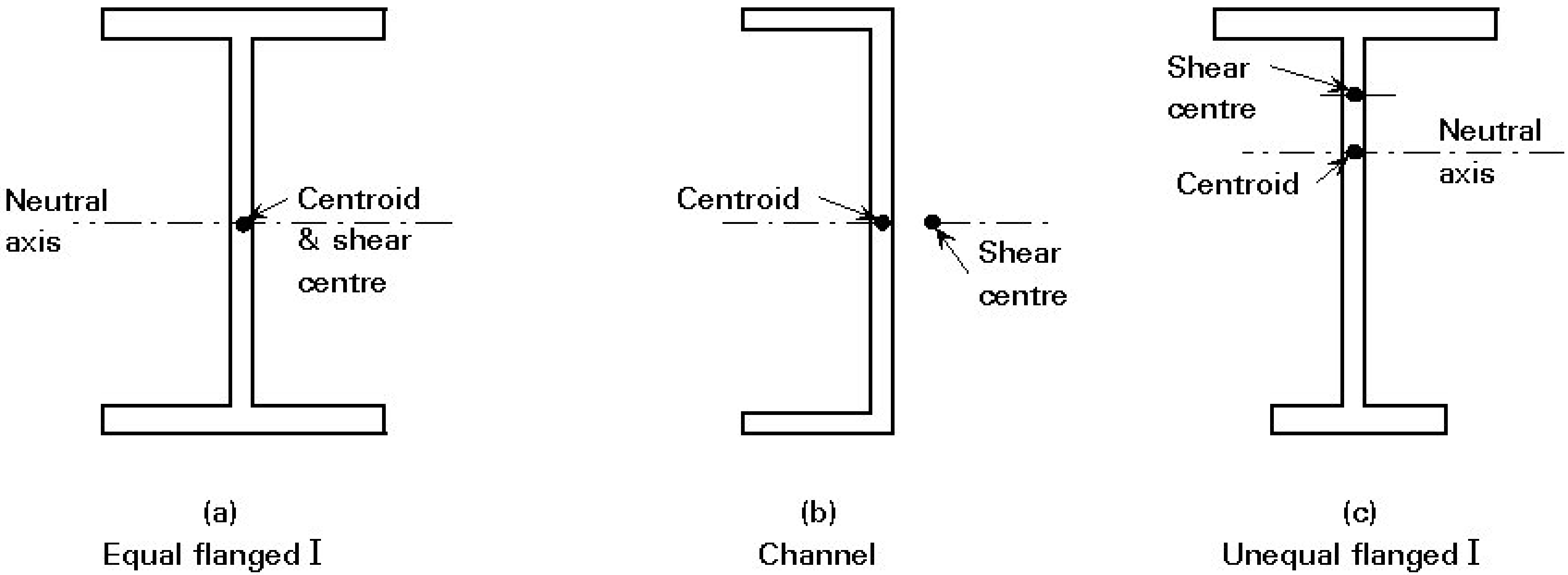
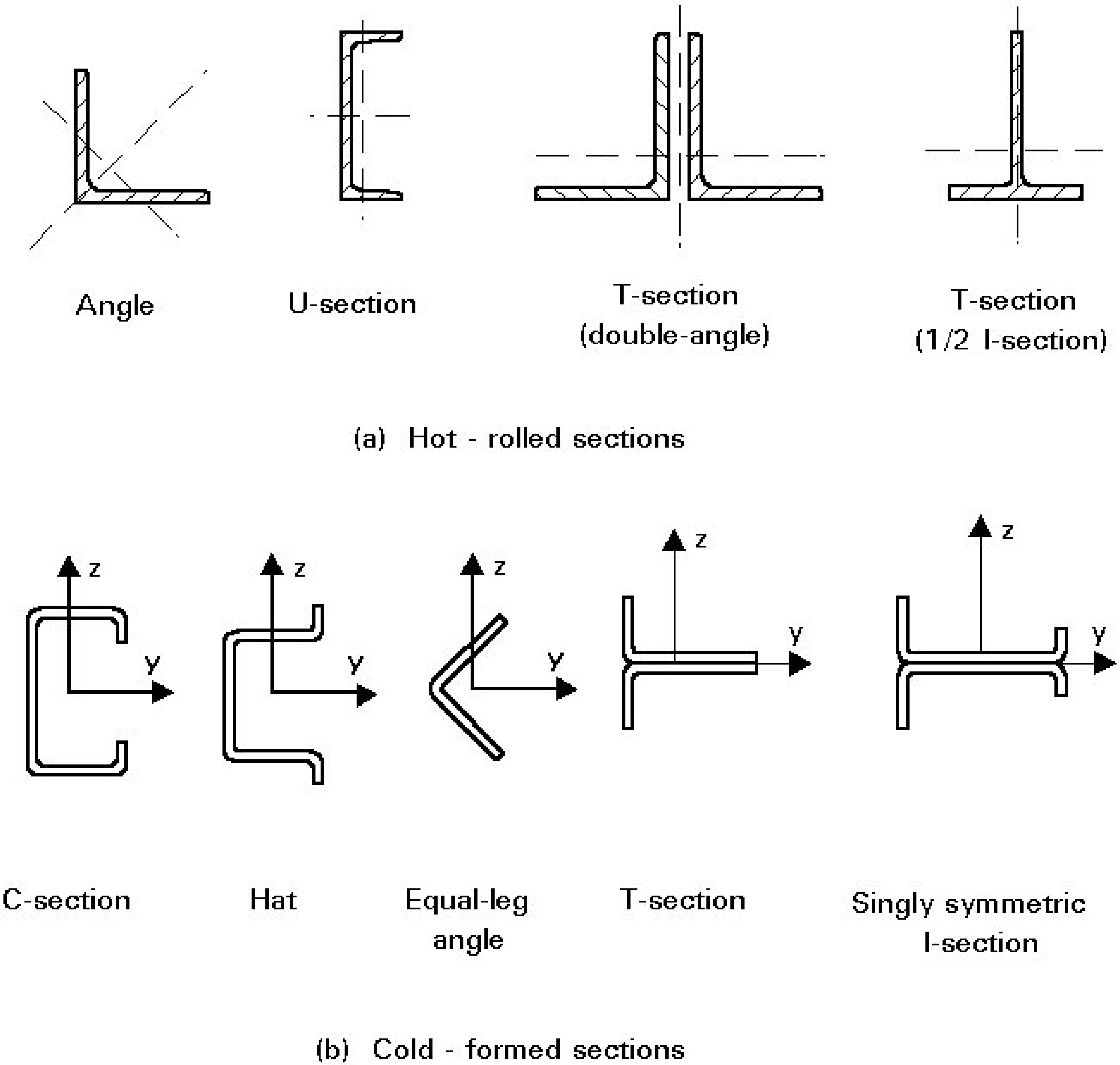


Figure 10 Equal flanged section and examples of sections with one axis of symmetry

Torsional-flexural buckling

- For of singly symmetric sections, about the major axis
- For unsymmetric sections, about any axis
- Rotation is always about shear centre



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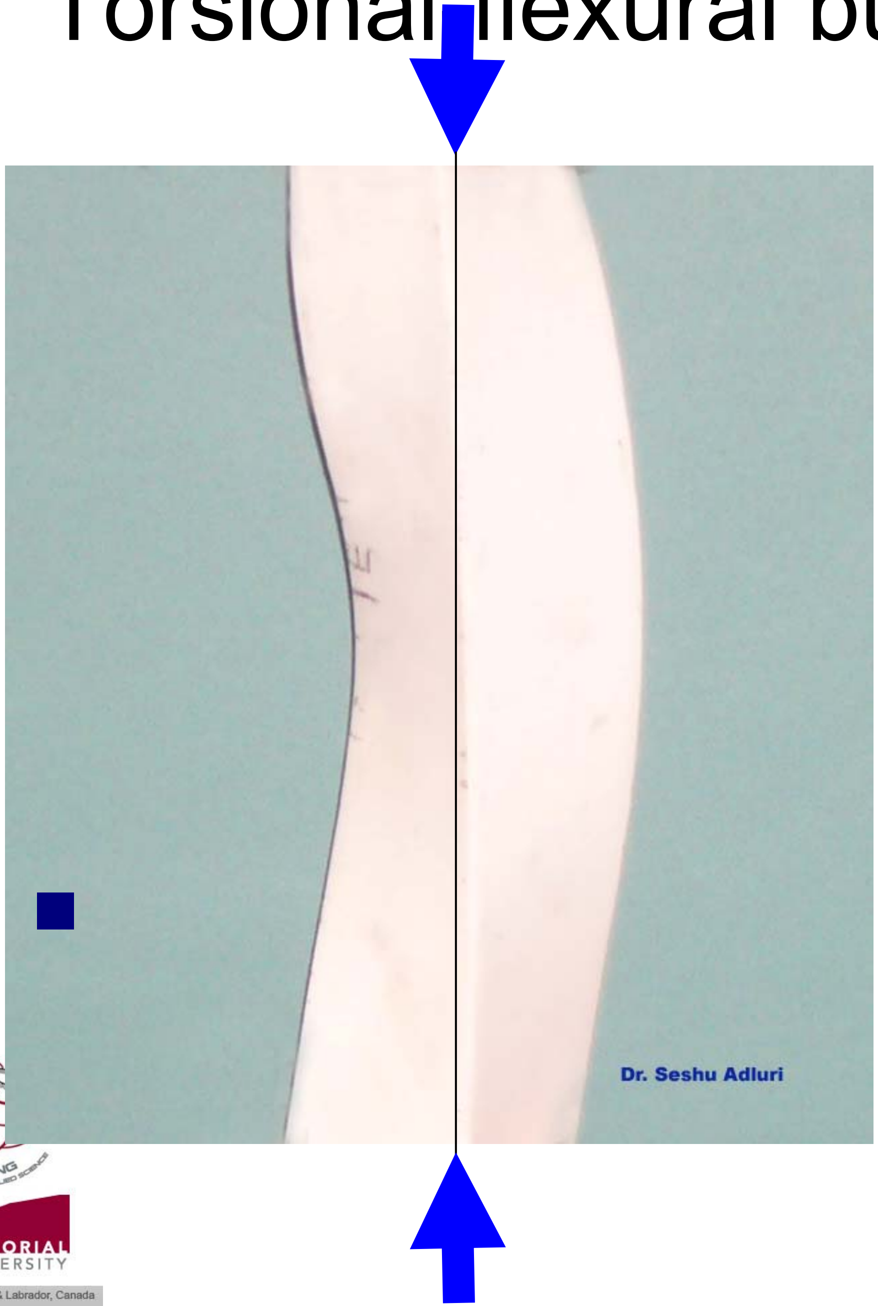
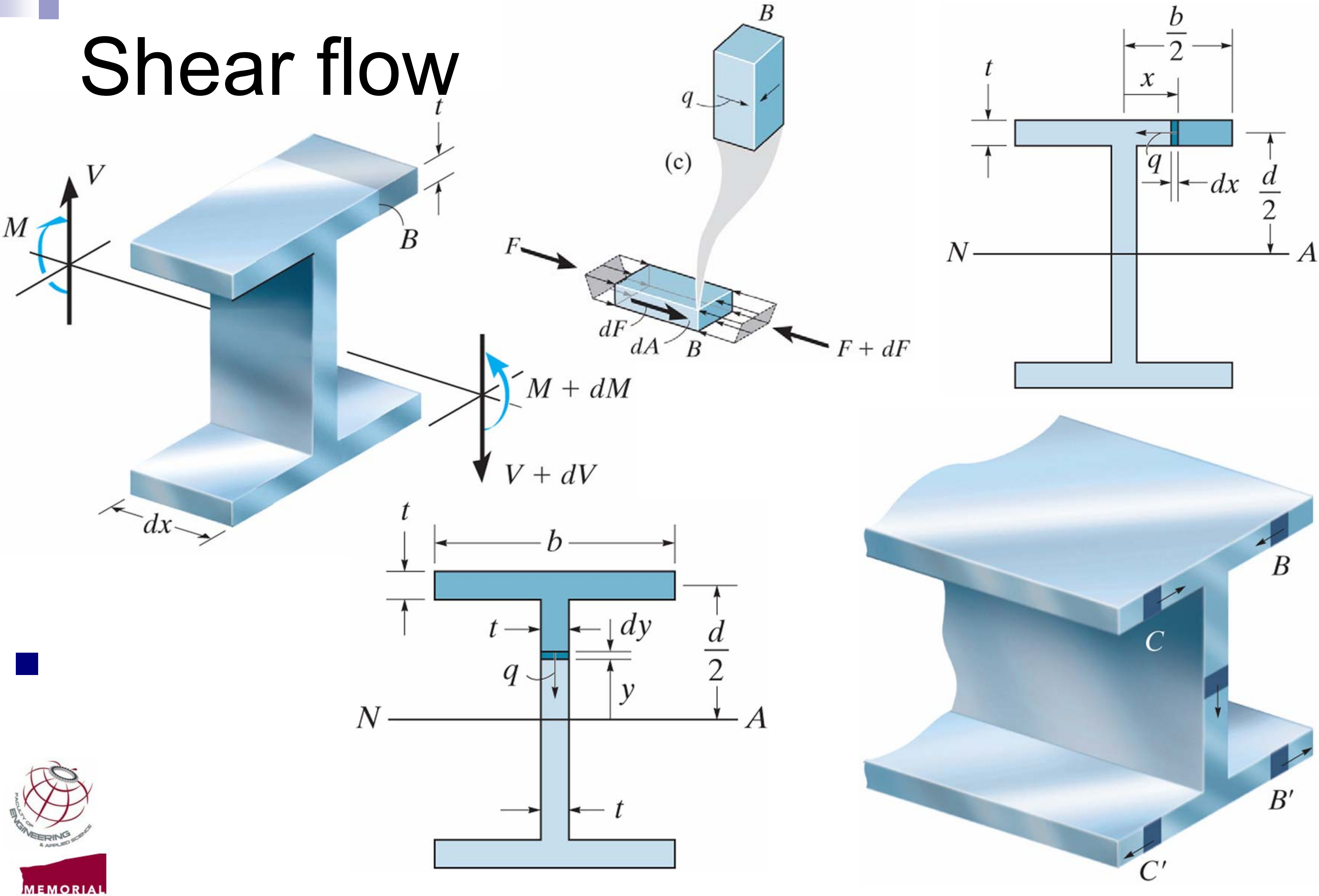
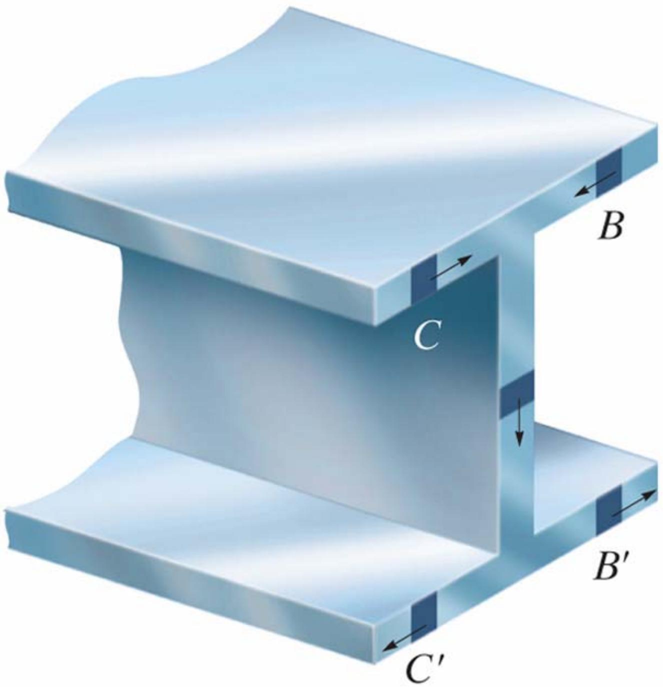
Torsional flexural buckling

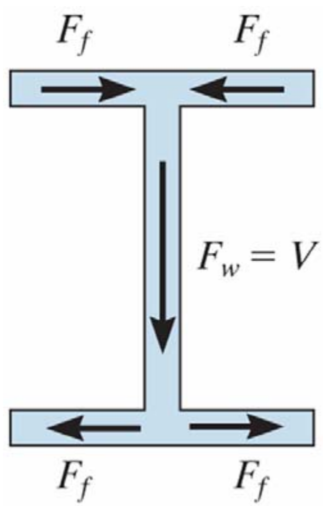


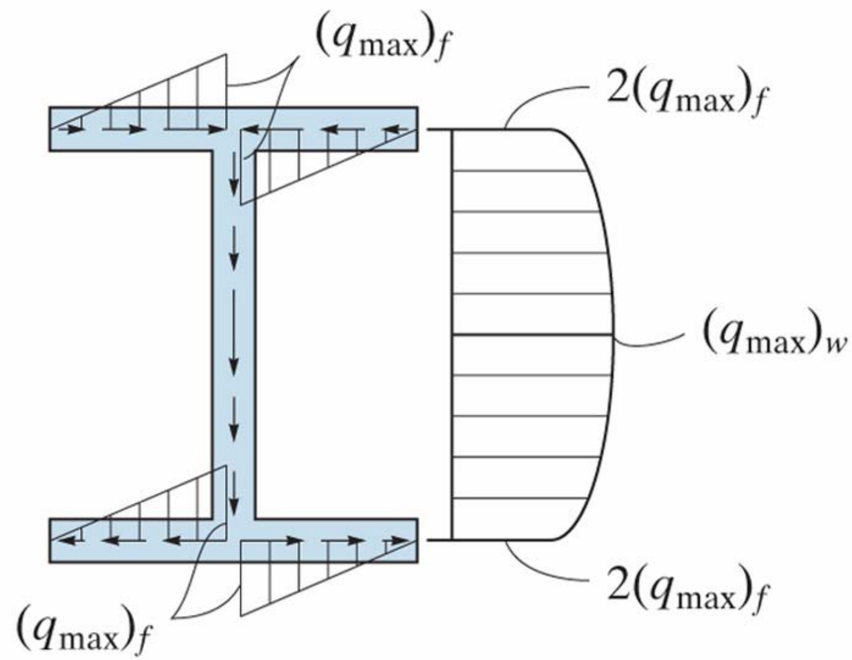
Shear flow



Shear flow

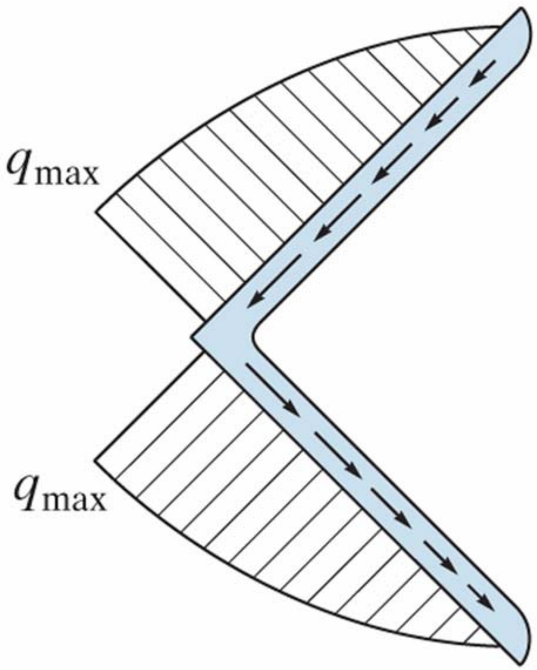






Shear-flow distribution

Shear flow



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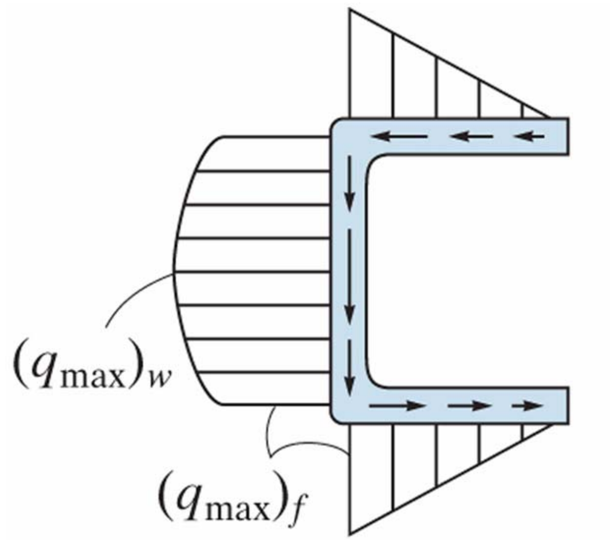
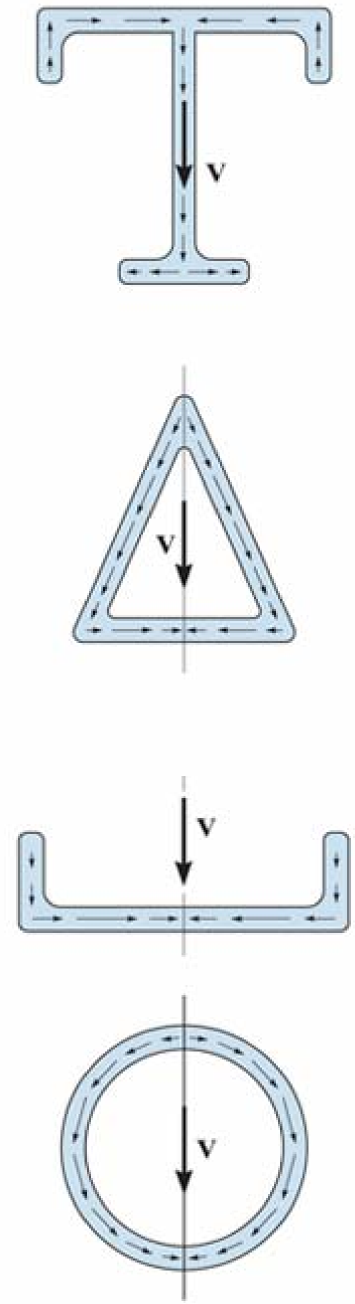
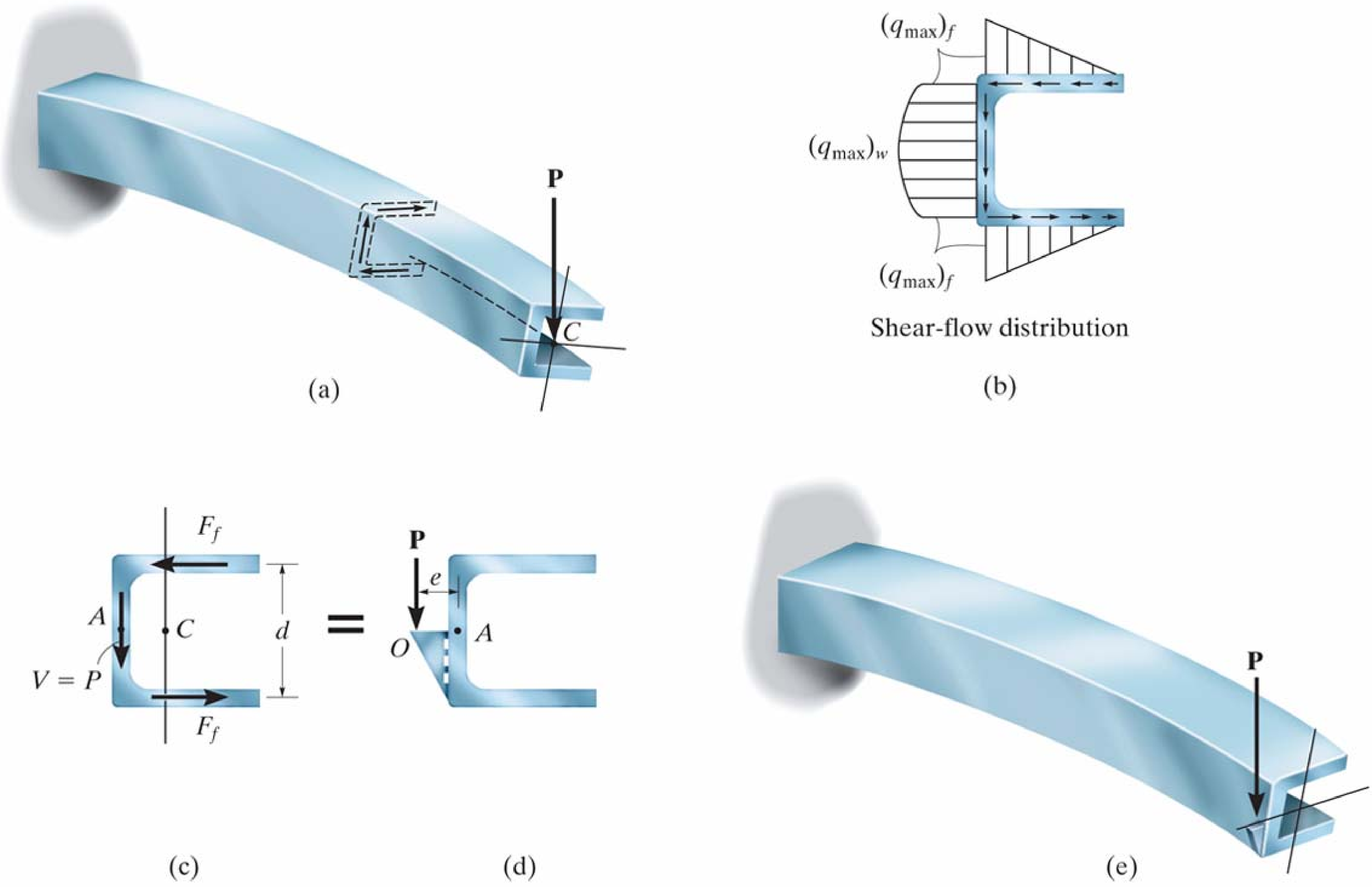


Shear flow distribution

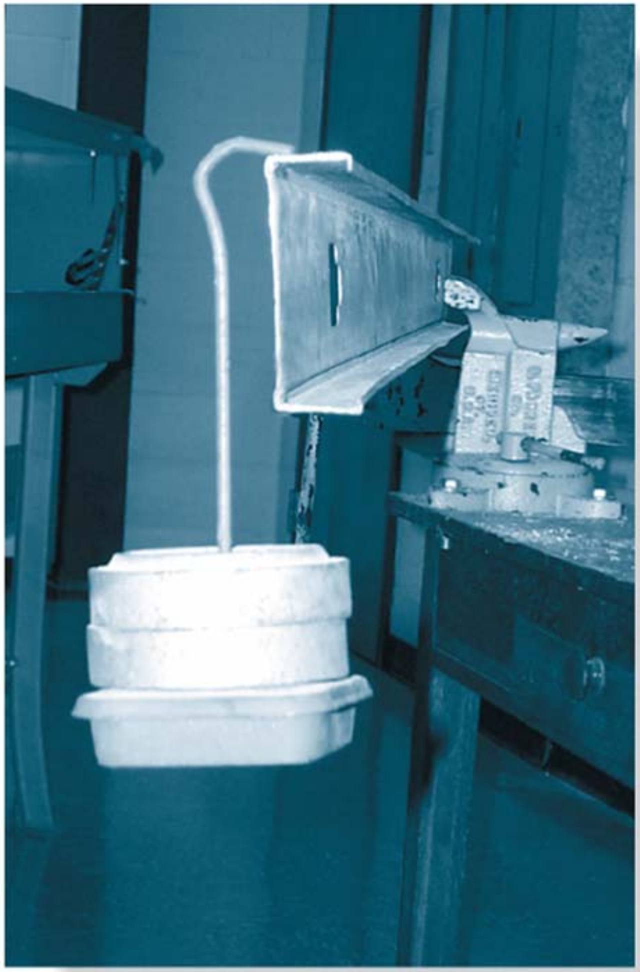


Shear flow q

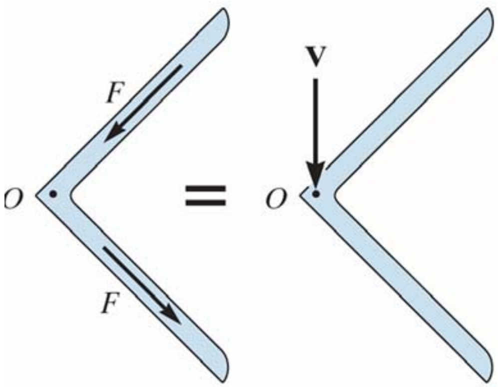
Shear centre

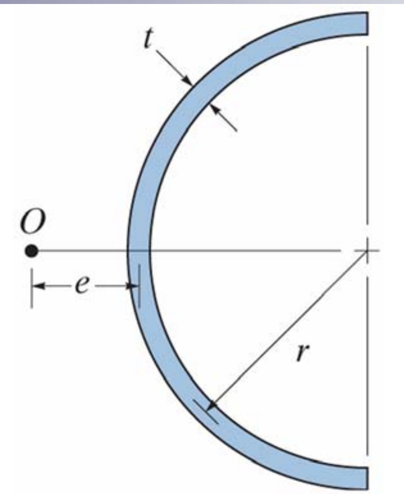
- 

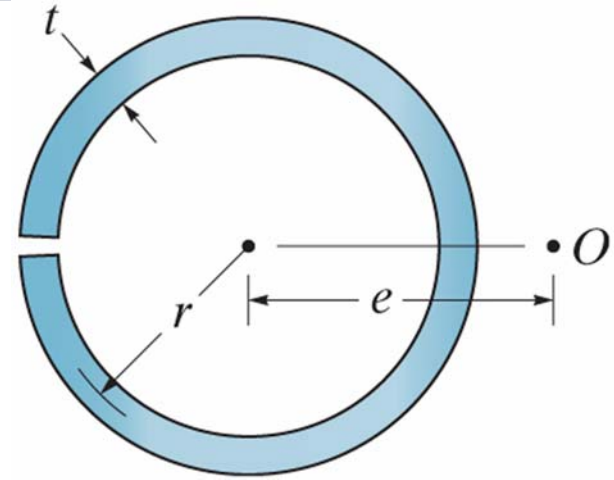
Shear flow effect

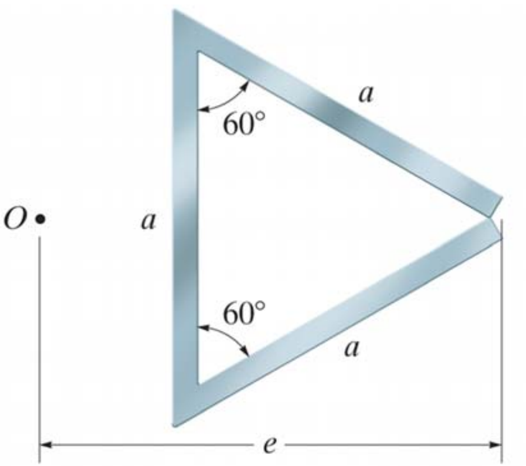


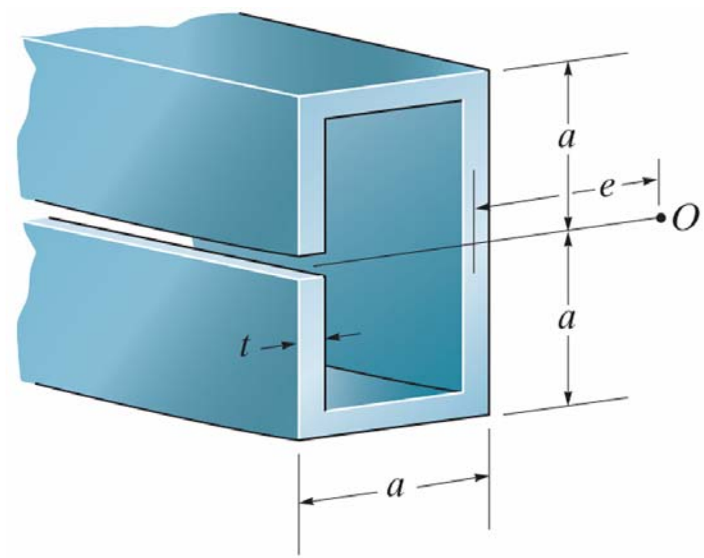
Shear centre

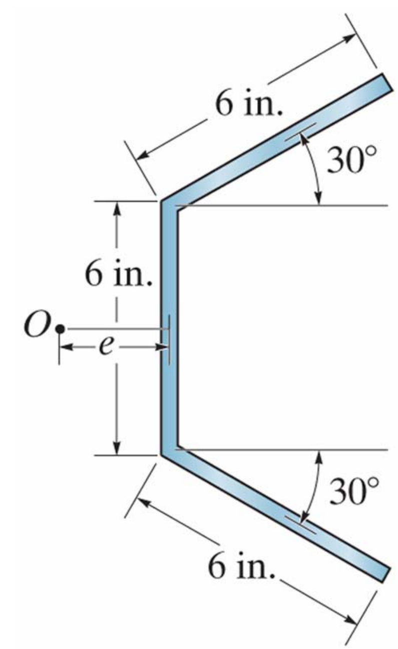






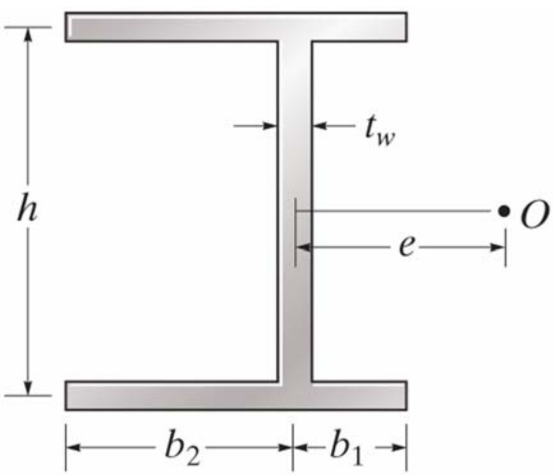


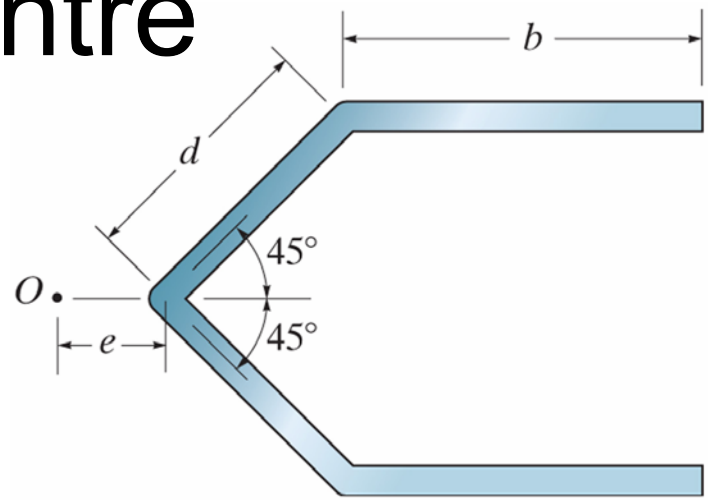


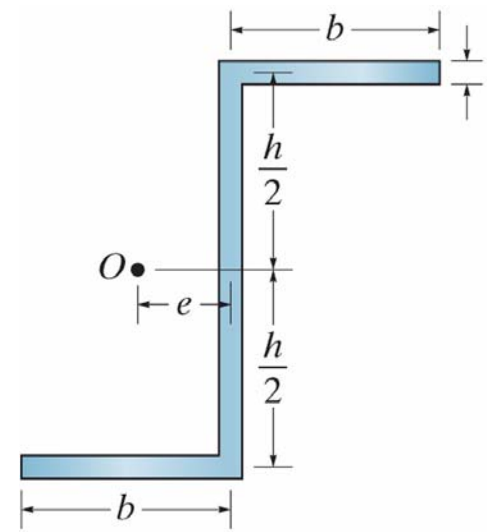


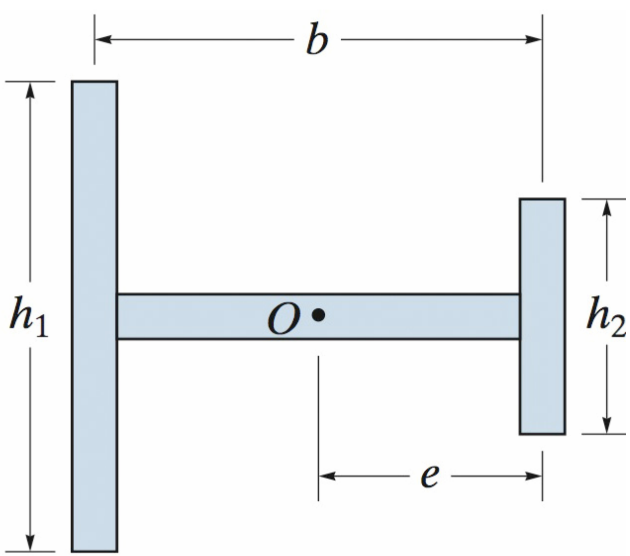
Newfoundland & Labrador, Canada

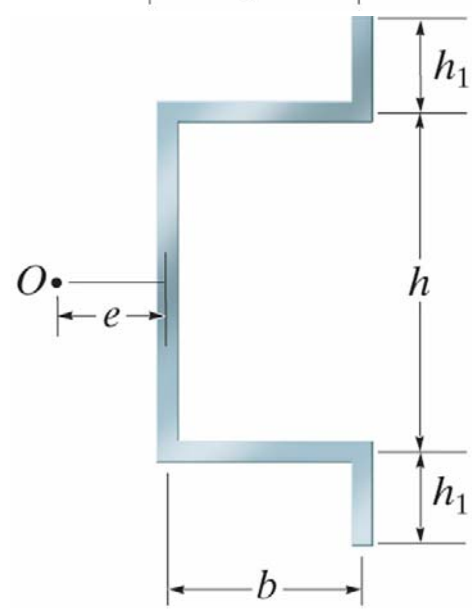
Shear centre











Local (Plate) buckling

-

Plate buckling

-

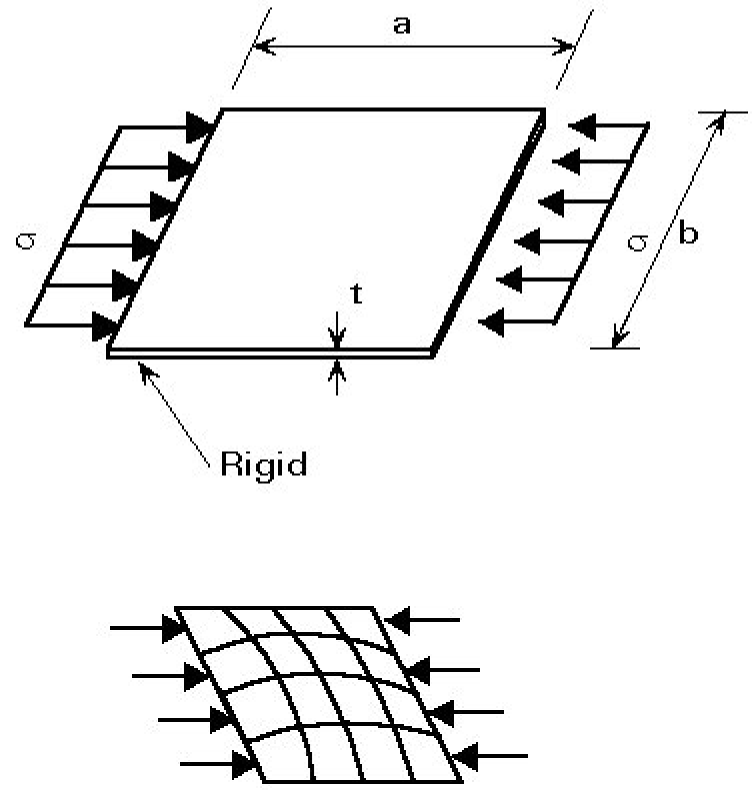


Figure 2 Fundamental case for compressive plate buckling

Plate buckling

- Effective width concept

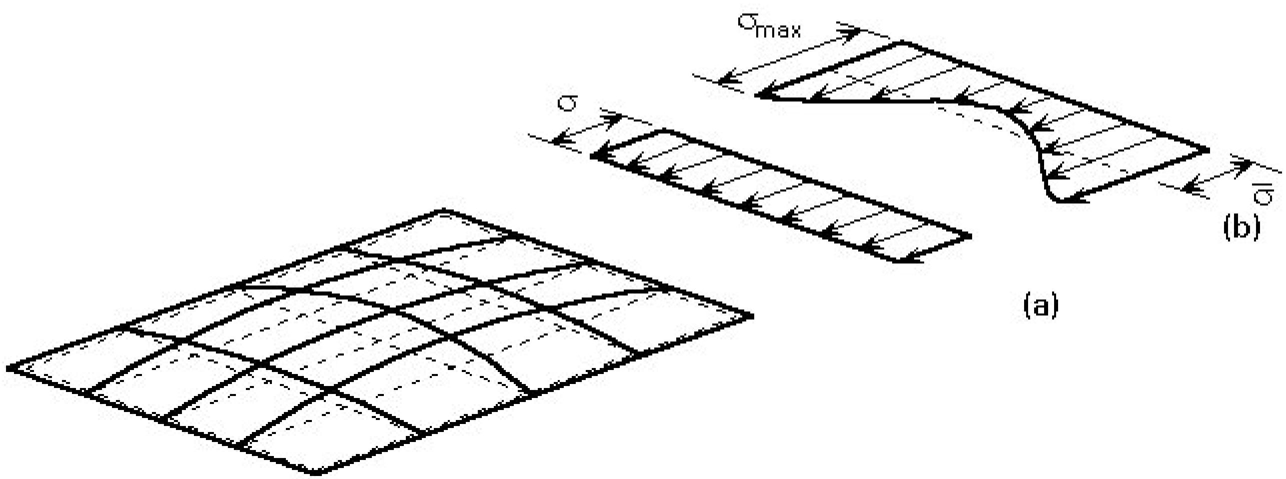
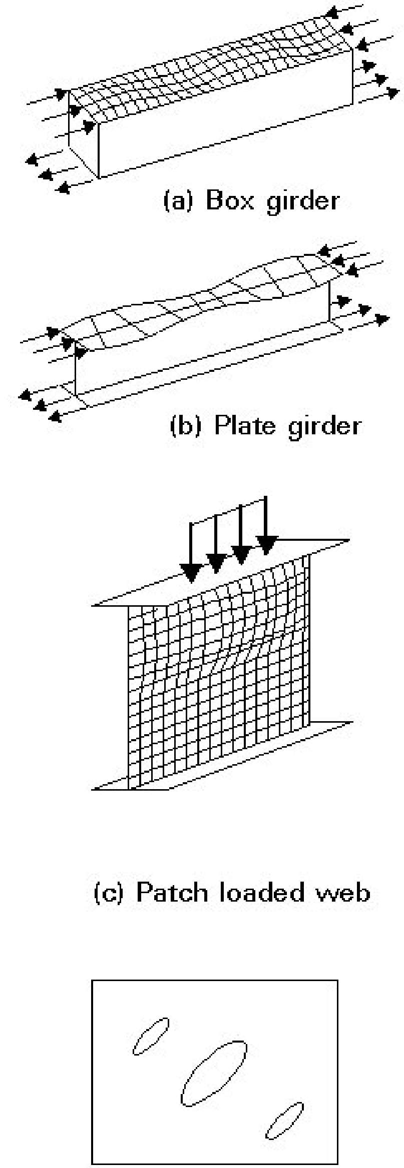


Figure 5 Stress distribution: (a) in the pre-buckling range and (b) in the post-buckling range

Plate buckling

- Different types of buckling depending on
  - b/t ratio
  - end conditions for plate segments
  - Table 1 for columns
  - Table 2 for beams and beam-columns



(a) Box girder

(b) Plate girder

(c) Patch loaded web

(d) Web subject to shear

Figure 1 Types of plate buckling

Web buckling

-

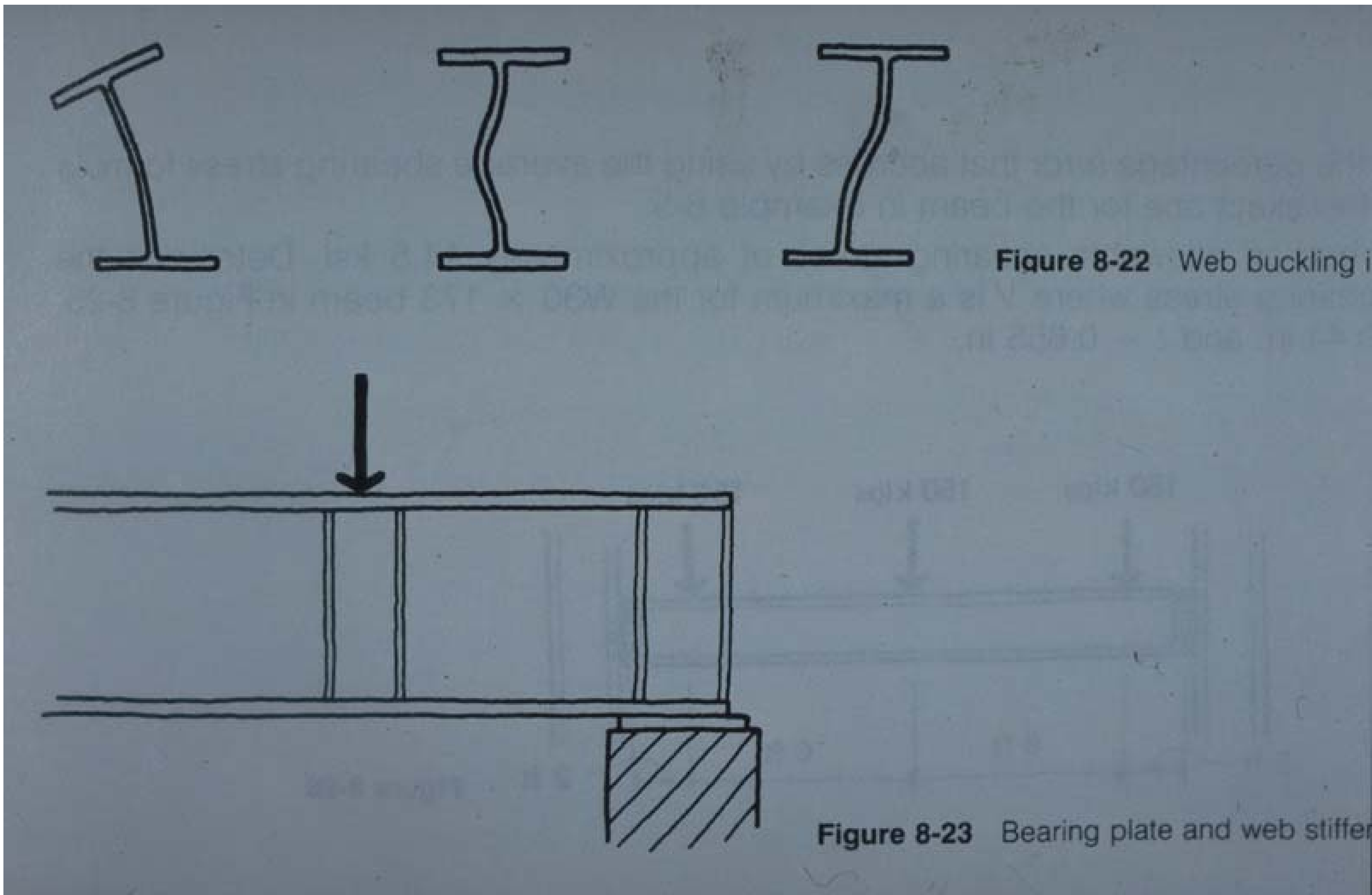


Plate buckling

- b/t ratio effect

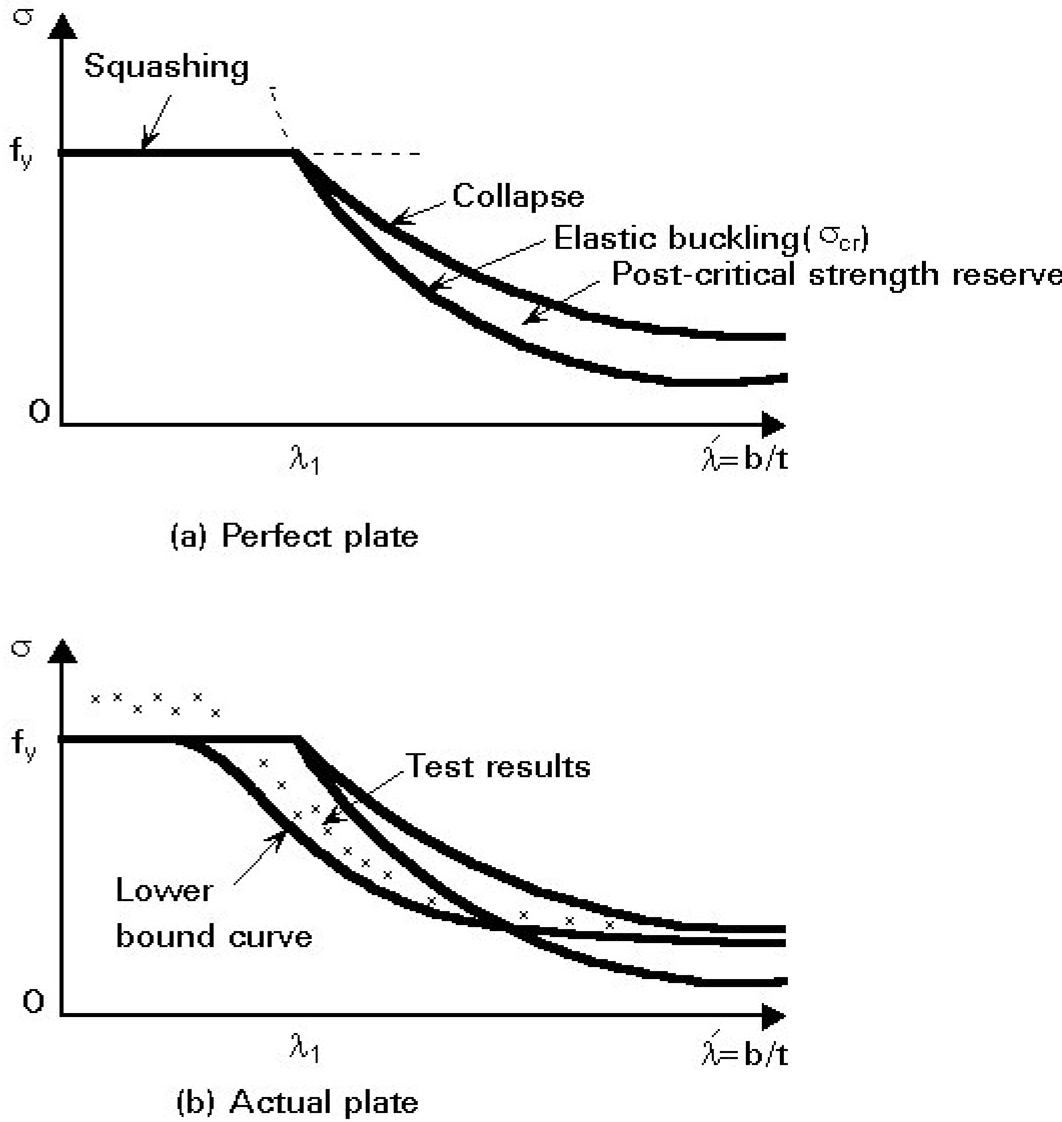
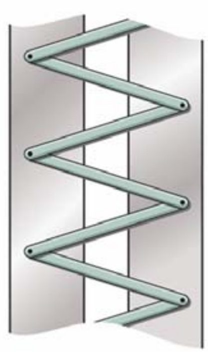
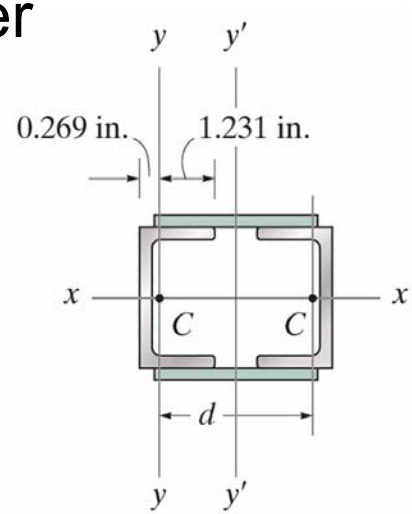


Figure 6 Influence of plate slenderness on buckling strength

Built-up columns

- Two or more sections
  - Stitch bolts
  - Batten plates
  - Lacing
  - Combined batten & lacing
  - Perforated cover plates





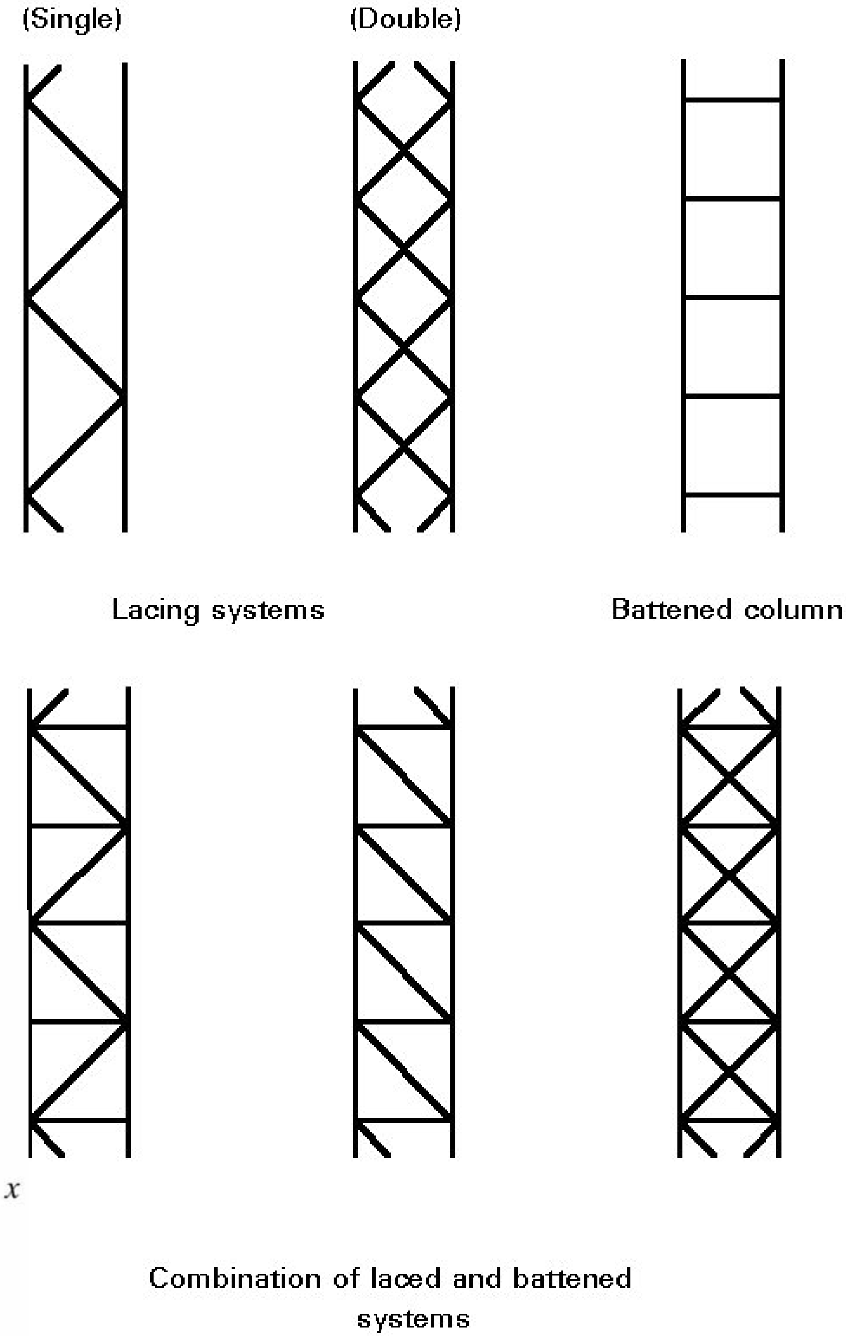
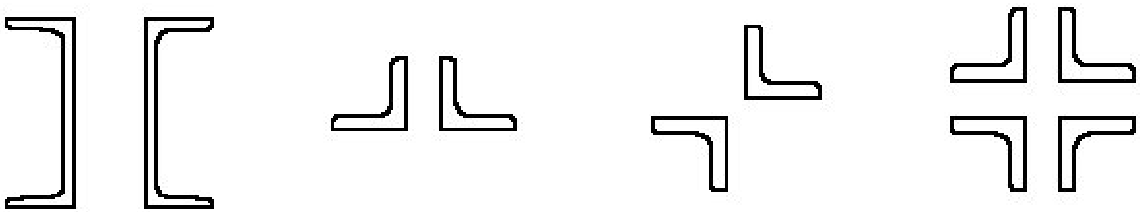
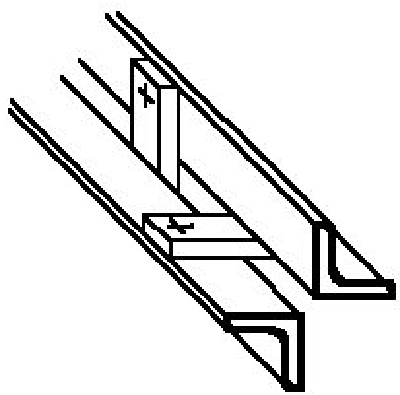


Figure 5 Laced and battened columns

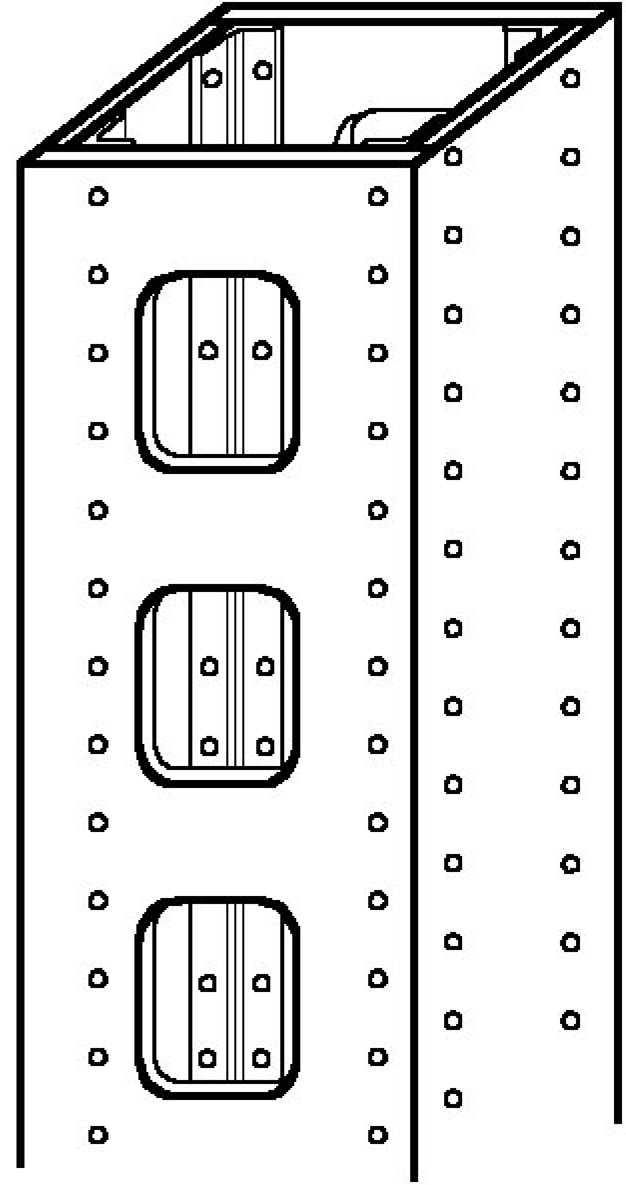
Built-up columns



Closely spaced built-up members



Detail of star-battened member

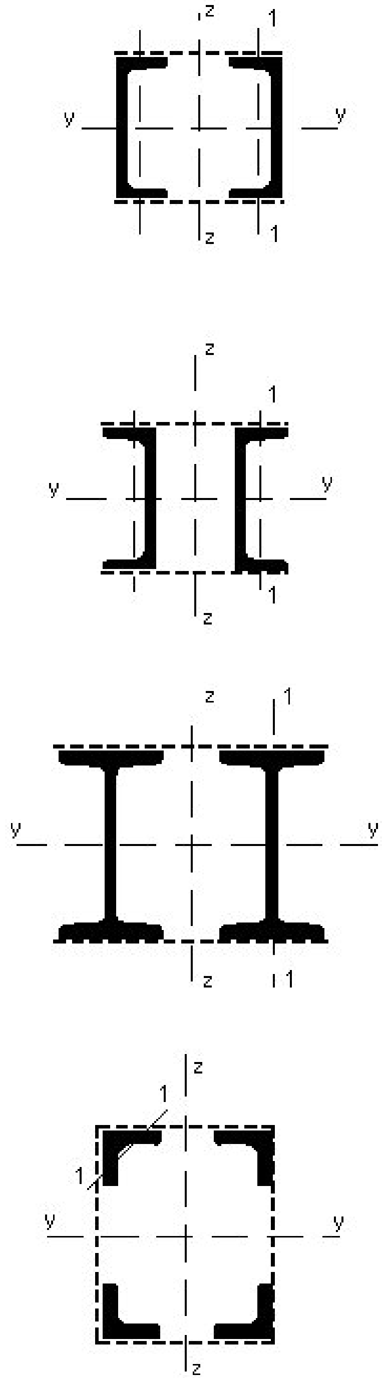


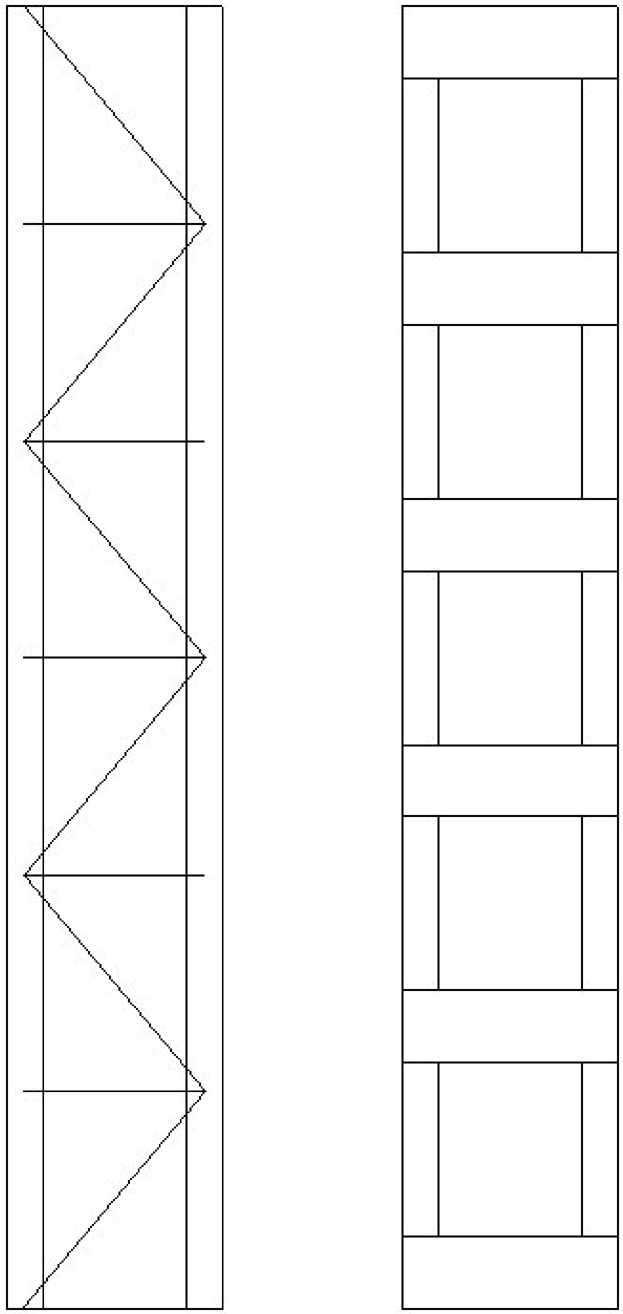
Perforated plate column

Built-up members

Built-up columns

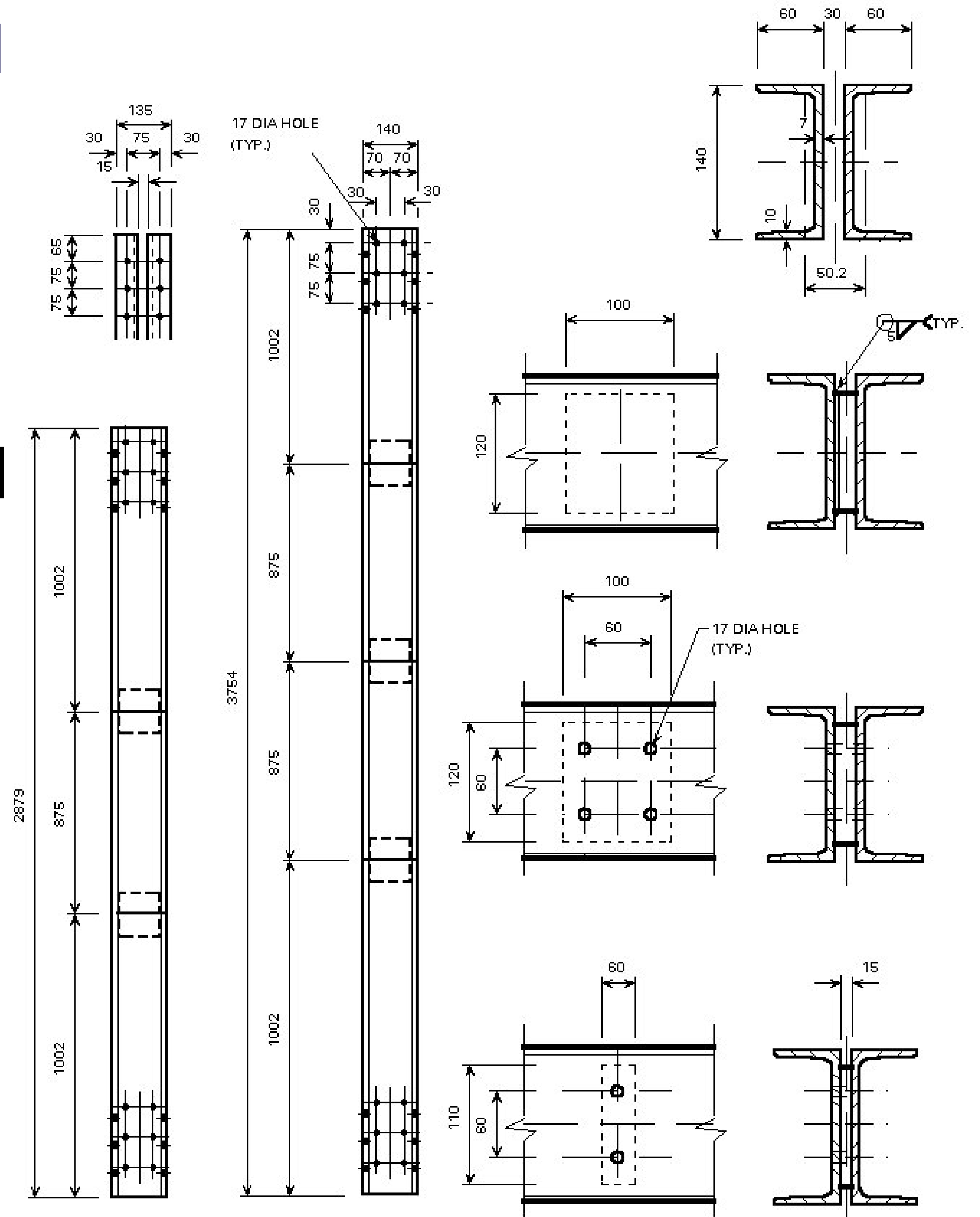
-





Built-up columns

- Closely spaced channels



Built-up columns

- Built-up member buckling is somewhat similar to frame buckling
  - Batten acts like beams
  - Battens get shear and moment due to the bending of the frame like built-up member at the time of buckling

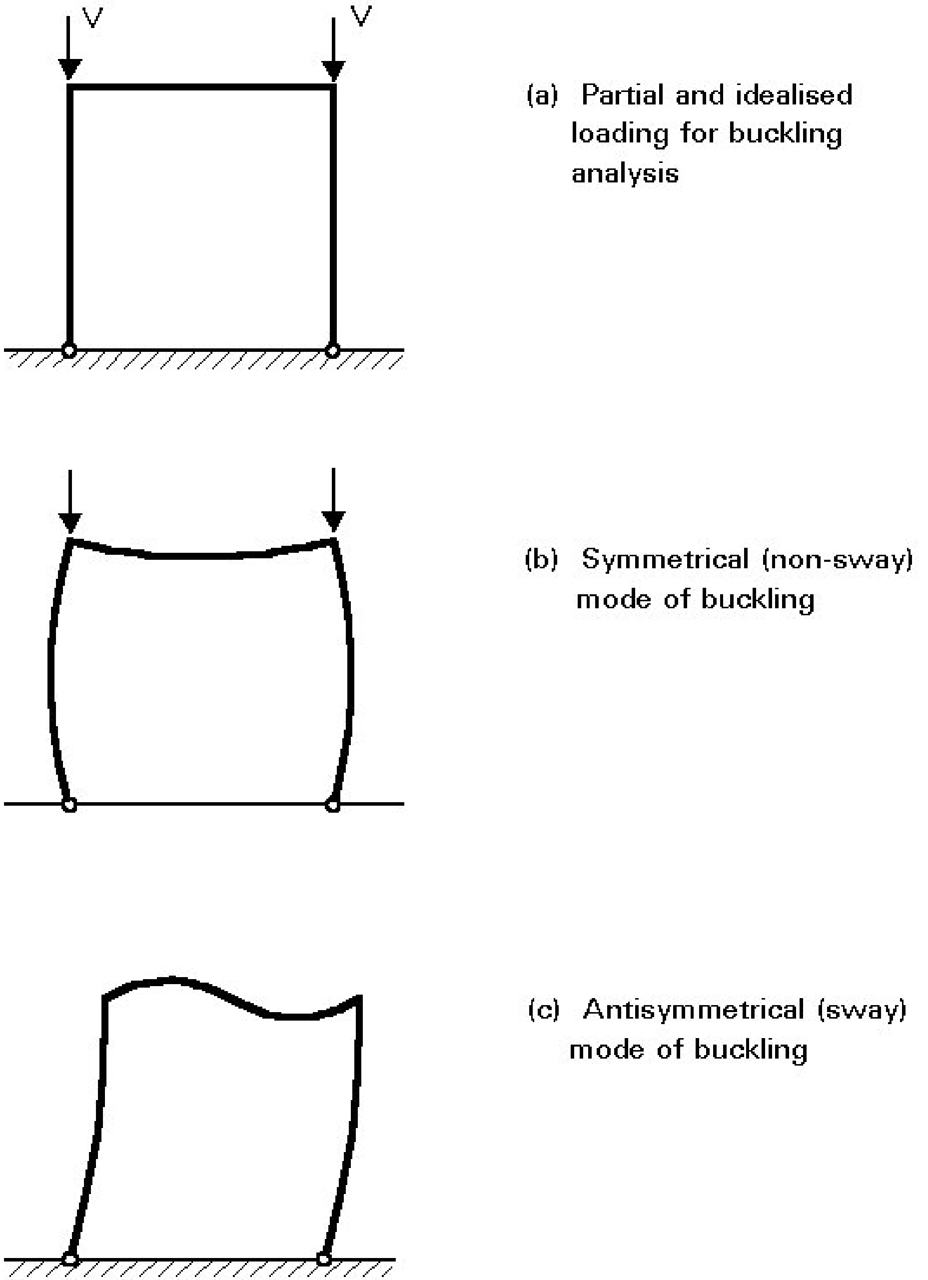


Figure 6 Buckling of frames

Battened column

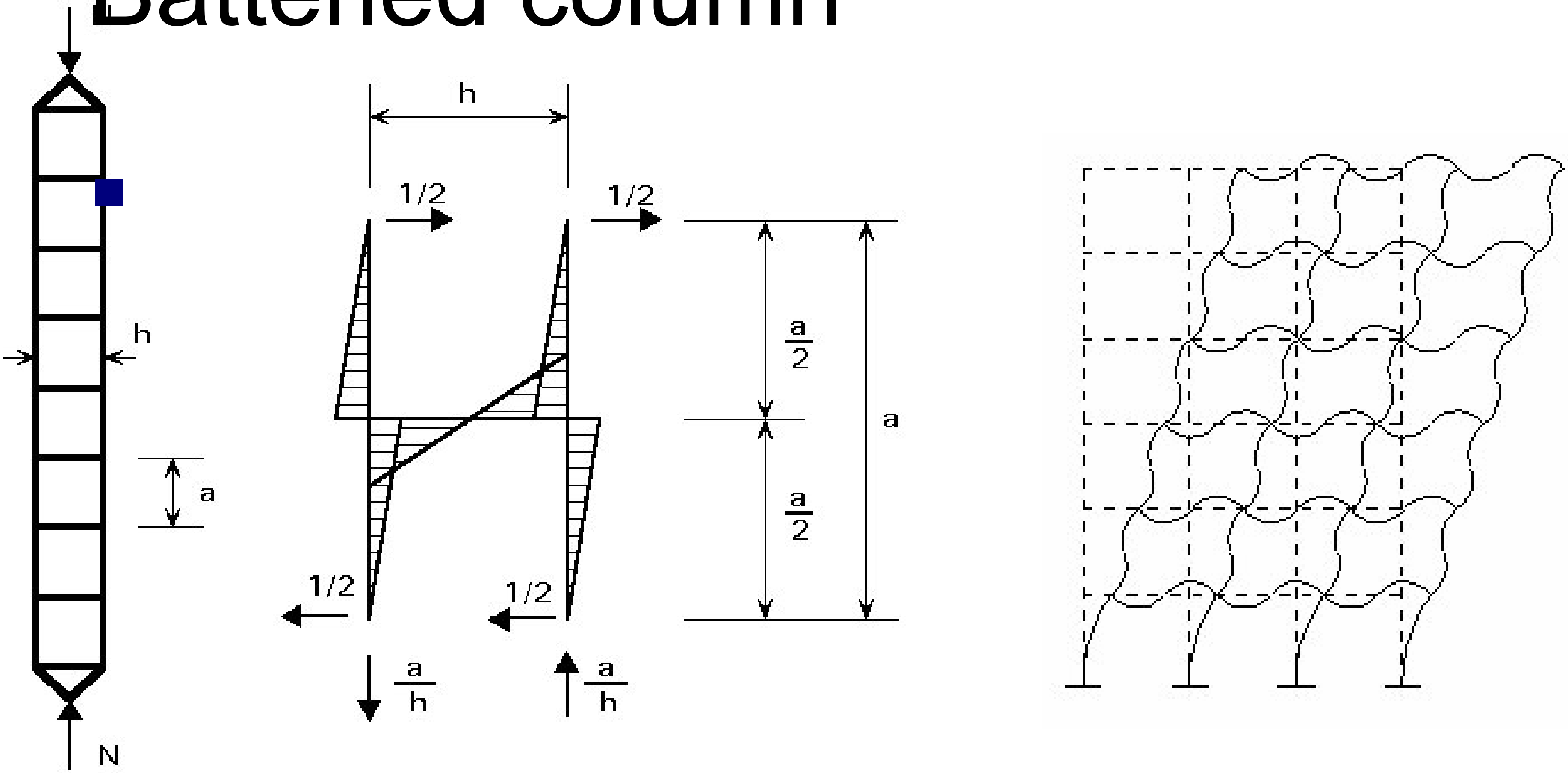
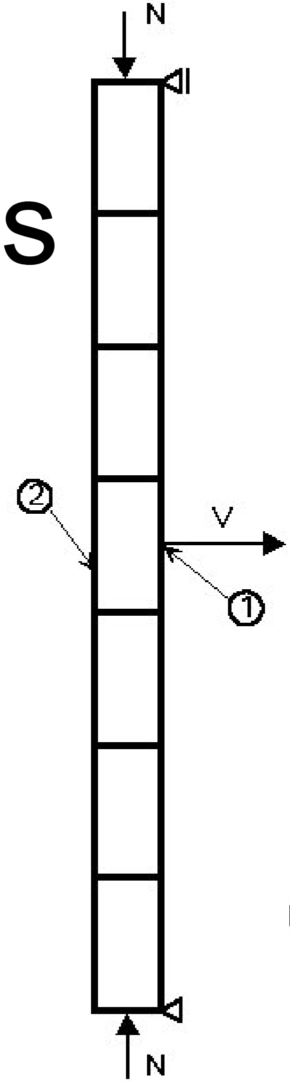
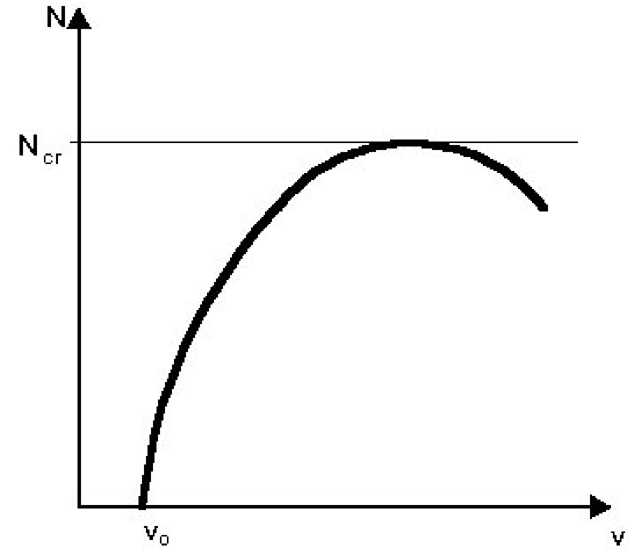


Figure 8 Battened built-up column

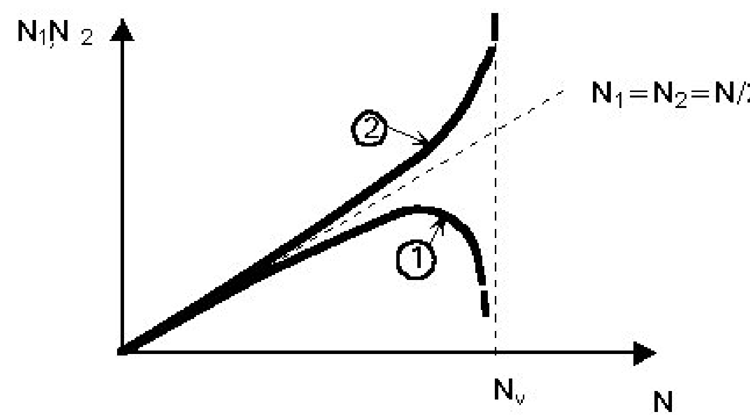
Built-up columns

- Design as per normal procedure
  - Moment of inertia about the axis which shifts due to the presence of gap needs parallel axis theorem
  - Effective slenderness ratio as per Cl. 19.1





(a)



(b)

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