UNITS AND DIMENSIONS

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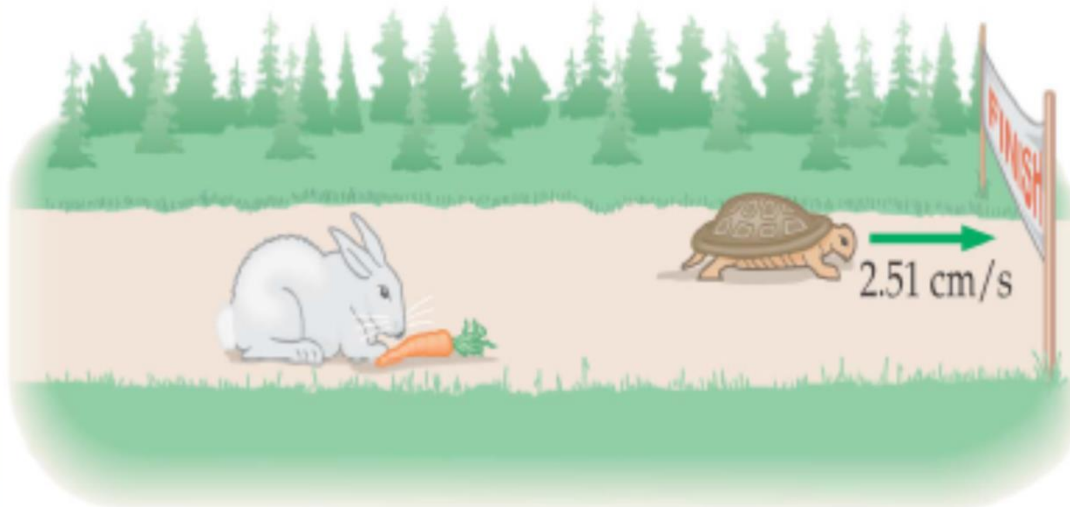
Measurement and Units

Fundamental units

Systems of units

Dimensional Analysis



UNITS AND DIMENSION ::

Why do we need units ?

We need units because we want to measure the Amount or quantity of some things.

To make this measurement globally acceptable we need to put some Unique measurement value.
This value is called a UNIT

Physical Quantities

- All things in classical mechanics can be expressed in terms of the **fundamental quantities:**
  - Length L
  - **Mass** M
  - Time T

Some examples of more complicated quantities:

- **Speed has the quantity of L / T (i.e. Kilometer per hour).**
- **Acceleration has the quantity of L/T^2**
- **Force has the quantity of ML/T^2 .**

Common Physical Quantities

Quantity	Dimension
Distance	[L]
Area	$[L^2]$
Volume	$[L^3]$
Velocity	[L]/[T]
Acceleration	$[L]/[T^2]$
Energy	$[M][L^2]/[T^2]$

Systems of Measurement

There are several units systems for measurement of physical quantities

The most common systems are **Metric , FPS, CGS and MKS** system

- For consistency, the **l'Systeme Internationale** (or SI) was adopted
  - The SI system is a special set of metric units

- **International System (SI) base units:**

Mass	*Kilogram*	*kg*
Length	*meter*	*m*
Time	*second*	*s*
Temperature	*Kelvin*	*K*
Current	*Ampere*	*A*

- All of the other SI units are derived from these base units
  - Examples of derived units:
    - **1 Newton = 1 N = 1 kg·m/s²**
    - **1 Joule = 1 J = 1 kg·m²/s²**
    - **1 Coulomb = 1 C = 1 A·s**

Systems of Unit

FPS: Foot, Pound, Second

CGS: Centimeter, Gram, and Second

MKS: Metre, Kilogram and second

SI: System International

Fundamental Units

Physical Unit	Symbol	Unit	Quantity symbol
length	l	metre	m
Mass	m	kilogram	kg
time	t	second	s
electric current	I	ampere	A
thermodynamic temperature	T	kelvin	K

Common Metric Prefixes

Prefix	Symbol	Meaning	Power of 10
Giga	G	1,000,000,000	10^9
Mega	M	1,000,000	10^6
kilo	k	1,000	10^3
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	m	0.000,001	10^{-6}
nano	n	0.000,000,001	10^{-9}

Using Metric prefixes:

- **1 mm = 1×10^{-3} m → 35 mm = 35×10^{-3} m or 3.5×10^{-2} m**
- **1 kg = 1×10^{3} g → 12 kg = 12×10^{3} g or 1.2×10^{4} g**

Exa	**E**	**10^{18}**
peta	P	10^{15}
Tera	T	10^{12}
hecta	h	10^{2}
Deka	da	10^{1}
Deci	d	10^{-1}
Pica	p	10^{-12}

unit
dimension } What's the difference?
value

What are Units?

Many people aren't sure of the difference. Let's try and get a set of definitions we can use.

Consider
110 mg of sodium
24 hands high
5 gal of gasoline

We'll break them up this way

Value	Unit	Dimension
110	mg	mass
24	hand	length
5	gal	volume (length3)

Systems of Unit

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Fundamental Units

Physical Unit	Symbol	Unit	Quantity symbol
length	l	metre	m
Mass	m	kilogram	kg
time	t	second	s
electric current	I	ampere	A
thermodynamic	T	kelvin	K
		temperature	

A "dimension" can be **measured or derived.** The "fundamental dimensions" (length, time, mass, temperature, amount) are distinct and are sufficient to define all the others.

We also use many derived dimensions (velocity, volume, density, etc.) for convenience.

"Units" can be **counted or measured**. Measured units are specific values of dimensions defined by law or custom.

Many different units can be used for a single dimension, as inches, miles, centimeters, furlongs, meters and Kilometer are all units used to measure the dimension length.

Derived Units

Physical Unit	Unit Symbol	Quantity
Acceleration	metre/second2	m/s2
Angular Velocity	radian/second	rad/s
Angular acceleration	radian/second2	rad/s2
Force	N or Newton	kgm/s2
Moment of Force	Newton metre	Nm
Work, Energy	Joule J or	Nm
Torque	Newton metre	Nm
Power	Watt	W= J/s2
Pressure	Pascal	Pa = N/m2
Frequency	Hertz	Hz or 1/s

Units and Calculations

It is always good practice to attach units to all numbers in an engineering calculation. Doing so

- attaches physical meaning to the numbers used,
- gives clues to methods for how the problem should be solved, and
- reduces the possibility of accidentally inverting part of the calculation.

Addition and Subtraction

Values MAY be added if UNITS are the same.
Values CANNOT be added if DIMENSIONS are different.

EXAMPLES:

$6ft + 4^\circ C = ???$

different dimensions: length, temperature -- so **cannot be added**

$6ft + 4in = ???$

$72in + 4in = 76in = 6.3ft$

same dimension: length, different units -- **can add**

Multiplication and Division

Values may be combined; units combine in similar fashion.

EXAMPLES:

$$12g \div 2ml = 6g/ml$$

$$2N \times 3m = 6N \cdot m$$

$$9^\circ C \div 2^\circ C = 4.5$$

4.5 is a "**dimensionless"** quantity (in this case a pure number)

You cannot cancel or lump units unless they are identical

Functions

Trigonometric functions can only have angular units (radians, degrees).

All other functions and function arguments, including exponentiation, powers, etc., must be dimensionless.

$(6m)^2 = (6\,m) \times (6\,m) = 36m^2$ is **OK;**

but $6^{(2m)} = ???$ is **meaningless**

Sin ($\frac{\pi}{2}$ m) is never defined.

Dimensional Homogeneity

Every valid equation must be "dimensionally homogeneous" (*a.k.a.* dimensionally consistent).

All additive terms must have the same dimension.

Dimensionless Quantities

When we say a quantity is dimensionless, we mean one of two things.

First, it may just be a number like we get when counting.

M ! mass, L ! length , T ! time
Dimensions of Some Common Mechanical Quantities

Quantity	Dimension	MKS unit
Angle	dimensionless	Dimensionless = radian
Steradian	dimensionless	Dimensionless = radi an^{2}
Area	L^{2}	m^{2}
Volume	L^{3}	m^{3}
Frequency	T^{-1}	s$^{!1}$ = hertz = Hz
Velocity	L !T^{-1}	m ! s $^{"1}$
Acceleration	L !T^{-2}	m ! s $^{"2}$
Angular Velocity	T^{-1}	rad ! s $^{"1}$
Angular Acceleration	T^{-2}	rad ! s $^{"2}$
Density	M ! L^{-3}	kg ! m $^{"3}$
Momentum	M ! L !T^{-1}	kg ! m ! s $^{"1}$
Angular Momentum	M ! L^{2} !T^{-1}	kg !m^{2} ! s $^{"1}$
Force	M ! L !T^{-2}	kg ◊m ◊s $^{-2}$ = newton = N
Work, Energy	M ! L^{2} !T^{-2}	kg !m^{2} ! s $^{"2}$ = joule = J
Torque	M ! L^{2} !T^{-2}	kg !m^{2} ! s $^{"2}$
Power	M ! L^{2} !T^{-3}	kg !m^{2} ! s $^{"3}$ = watt = W
Pressure	M ! L^{-1} !T^{-2}	kg ! m $^{"1}$! s $^{"2}$ = pascal= Pa

Arithmetic and Dimensions

There are strict rules for doing arithmetic with quantities that have dimension.

1. You can only add, subtract, or compare quantities with the same dimension.
So, you can add two lengths, or add two masses, but you can’t add a length and a mass.
2. You can multiply and divide quantities with any dimension. Anything goes with multiplication and dimension.
3. Sine, cosines, logarithms, etc. The input x in something like sin(x) or ln(x) or log(x) must always be dimensionless and unit less. No exceptions.
4. Exponentiation, for instance mb, involves two quantities: an exponent (b) and a base (m).

• The exponent must be dimensionless. That is, it must be a pure number with no units. No exceptions.

• The base must also be dimensionless, unless the exponent happens to be an integer. Or, more precisely, the dimensions of the base have to match the exponent in a way that the result has sensible dimensions.

So, $(3\text{miles})^2$ is a perfectly sensible calculation, producing a result of 9 square miles. But $\sqrt{3\text{miles}}$ is meaningless.

That business above starting, “Or, more precisely ...” means that it would be fine, for example, to take the square-root of “9 square miles.”

The result would be “3 miles.” This will be clearer when you’ve read about dimensions of calculated quantities.